# Generalized Neuron Based Digital Communication Channel Equalizer

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#### Abstract

Equalization is necessary in digital communication system to mitigate the effect of intersymbol interference (ISI) and other nonlinear distortions. In order to reduce complexity the application of generalized neuron (GN) to adaptive channel equalization in a digital communication system with duo-binary signals is investigated. It uses only a single GN thus there is no problem of selection of initial architecture of the neural network giving optimum performance. Low complexity and fast convergence characteristic of GN based equalizer make it suitable for real time application. Bit error rate (BER) over a wide range of signal to noise ratio (SNR) is noted. It has been shown that BER performance approaches to optimal Bayesian solution.

**Keywords:** Digital communication, ISI, Channel equalizer, ANN, GN.

## 1. Introduction

BAND-limited high speed digital transmission suffers from ISI and various other noise sources. Nonlinear

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<sup>2</sup>Professor, Electronics & telecommunication Engineering and Computer Engineering, Military College of telecommunication Engineering, Mhow, India. Email: drbkjoshi@gmail.com distortion is a significant factor hindering further increase in attainable data rate. Equalization is necessary at the receiver to overcome these channel impairments [1]. Since communication channels are time varying in nature hence adaptive equalization is required. Figure 1 shows the simplified model of a discrete time transmission model of a digital communication system. Figure 2 shows the non-linear channel model. NL represents the nonlinearities involved.

s[k], is the original sequence to be transmitted, where k is any time instant. The block channel represents the combined response of the transmitting filter, transmission media and RF/IF sections of the receiver filter. q[k], is the additive white Gaussian noise (AWGN) that corrupts

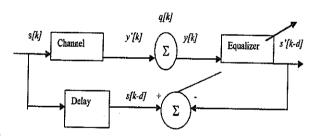


Figure 1: Simplified Block Diagram Of Discrete
Time Transmission Model

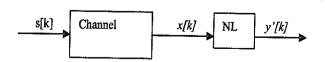


Figure 2: Nonlinear Channel Model

the channel output y'[k]. The channel output y'[k] corrupted with AWGN q[k] forms the input to the equalizer y[k]. The equalizer at the receiver makes an estimate s'[k-d], of the expected delayed transmitted sequence s[k-d], from the knowledge of channel output y[k], present and past values. Where d, is equalizer delay. The difference in the estimated sequence and expected delayed transmitted sequence is minimized during training of the equalizer periodically.

Band-limited communication channels are generally modeled as digital FIR filters represented as

$$H(z) = \sum_{i=0}^{N} hiz^{-i} \tag{1}$$

Where, N is the channel order.

The channel output vector can be represented as

$$y(k) = [y(k), y(k-1), ..., y(k-m+1)]^T$$
 (2)

Where, m is the order of equalizer.

Conventionally, equalization has been considered as a deconvolution problem, where a finite impulse response filter (FIR) based linear transverse equalizer (LTE) is used to invert the channel response and its parameters are adjusted using minimum mean square error (MMSE) criterion [1]. Least mean square error (LMS) algorithm is mostly used to tune the tap weights of the filters to minimize error iteratively. Linear filter based equalizers performance degrades under severe nonlinear distortion conditions. The maximum likelihood sequence estimation (MLSE) [2] requires batch processing of the entire received sequence, gives nearly optimum results but its very high computational complexity restricts its use hence practically not suitable and makes symbol by symbol detection as a good option. The equalization can be treated as a classification problem where, the job of an equalizer is to assign the received signal to one of the signal constellation. The optimal solution (least misclassification) to this classification problem is given by Bay's theory. The channel input vector for an  $m^{th}$  order equalizer is given by

$$s(k) = [s(k), s(k-1), s(k-2), \dots, s(k-m+1-N)]^T$$
 (3)

And can take  $Ns = 2^{N+m}$ , different values.

Different, Ns possible values of noiseless channel output vector is given by

$$y'(k) = [y'(k), y'(k-1), \dots, y'(k-m+1)]^T$$
 (4)

Which, is to be divided into two classes

$$Y^{+} = \{y'(k) \mid s(k-d) = 1\}$$

$$Y^{-} = \{y'(k) \mid s(k-d) = -1\}$$
(5)

y(k) is a random process having conditional Gaussian density functions centered at each of the  $Y_i^+$  and  $Y_i^-$ , where i  $\in \{1,2,\ldots,Ns/2\}$ .

If the transmitted sequence s(k) is an independent identically distributed (i. i. d.) and equi-probable binary sequence with values  $\{+1, -1\}$ . For this sequence the optimal solution to this classification task is given by Bay's theory as [3]

$$s'(k-d) = \operatorname{sgn}(f_B(y(k))) = \begin{cases} +1 & f_B(y(k)) \ge 0\\ -1 & f_B(y(k)) < 0 \end{cases}$$
(6)

$$f_{B}(k) = \sum_{i} \exp\left(-\frac{\|y(k) - Y_{i}^{+}\|^{2}}{2\sigma_{n}^{2}}\right)$$
$$-\sum_{j} \exp\left(-\frac{\|y(k) - Y_{j}^{-}\|^{2}}{2\sigma_{n}^{2}}\right)$$
(7)

Where  $\sigma_n^2$  is the variance of the AWGN. Good equalizers for this channel will than approximate the function given by (7) as a decision boundary for classification, which is a nonlinear function. Bayesian equalizer provides the lower performance bound for symbol-by-symbol equalizers in terms of probability of error or BER. Nonlinear mapping capability of artificial neural network (ANN) and fuzzy logic make them a suitable choice for nonlinear equalization. Several equalizers are developed in the past to address this problem using ANN and fuzzy logic. Some of them are reported in [3-11]. Such structures usually outperform LTE and also compensate for nonlinearities in the channel with varying degree of success. ANN based equalizers work even if the channel is unknown while fuzzy equalizers require that the channel must be known a prior. The major problem of such equalizers is the high complexity and computational requirement which can further be reduced. An integration of ANNs and fuzzy set theoretic approach will offer advantages of both the techniques.

The common neuron model has been modified to obtain a generalized neuron (GN) model using fuzzy compensatory operators to reduce the complexity of the structure and overcome the problems such as initial selection of architecture of neural network giving optimum performance for complex function mapping, which affects the training time requirement and also fault tolerant capabilities of the ANN [12]. This neuron provides flexibility and fault tolerant capability to cope up with the nonlinearities involved in the system. GN has been used successfully for power systems problems [13-14]. Application of this significantly reduced complexity GN as a channel equalizer in digital communication systems is demonstrated in this paper.

There have been introduced many different nonlinear devices models and channel models, so a unitary comparison between all known equalizers is difficult to be done. Bayesian solution provides the minimum average BER achievable for symbol decision and indirect—modeling equalizer structure. It has been shown that BER of proposed equalizer outperforms conventional LTE LMS equalizer approaches to that of optimal Bayesian equalizer under linear and non-linear conditions. The proposed equalizer provides acceptable training and BER performance.

# 2. GENERALIZED NEURON MODEL

Existing conventional neuron model generally uses an aggregation function and its transformation through an activation function. Generally summation is used as aggregation and sigmoid, radial basis, tangent hyperbolic or linear limiters etc. as activation function. Generalized neuron structure shown in figure 3 is developed by modifying the conventional neuron structure using fuzzy compensatory aggregation operators along with fuzzy activation functions. Aggregation operation in GN is performed partly by sum ( $\Sigma$ ,) and partly by product ( $\Pi$ ) functions. Bipolar sigmoid function (f1) is used as a transformation function for  $\Sigma_i$  part and Gaussian function (f2) is used as transformation function for  $\Pi$  part of the structure. The final output is the summation of the  $\Sigma_1$ output and  $\Pi$  output with weight sharing as W and (1-W)respectively. The input output relationship for GN is given by following expressions.

Let  $Y_i$  represents the input vector to the equalizer which is y[k] as given by equation (2).

The  $\Sigma_1$  part output of GN is calculated as

$$Os = 2 * s \_out - 1 \tag{8}$$

Where, 
$$s_out = \frac{1}{1+e^{-\lambda s^*sum_s}}$$
, (9)

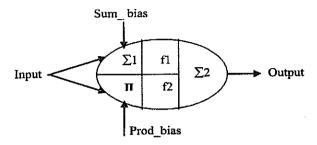


Figure 3: Structure of GN

 $\mathcal{A}_{\mathcal{S}}$  , is the gain scale factor for  $\boldsymbol{\Sigma}_{_{1}}$  part of GN,

$$sum\_s = \sum Wsi\ Yi + Xos$$

Xos, is the bias to  $\Sigma_1$  part and Wsi, is the weight vector.

The II part output of GN is calculated as

$$Op = e^{-\lambda p * prod} - p^2 \tag{10}$$

Where,  $\lambda p$  is the gain scale factor for  $\Pi$  part of the GN.

$$prod _p = \prod WpiYi * Xop$$

 $X_{OP}$ , is the bias to  $\Pi$  part and  $W_{Pi}$ , is the weight vector.

The final output is

$$Oi = W * Os + (1 - W) * Op$$
 (11)

Where, W is the weight vector.  $O_i$  is the estimated output vector s'[k-d].

#### 3. Learning Algorithm Of Gn

Back propagation learning algorithm is used to train the network.

Following steps are to be followed to train the network till the mean square error reaches to minimum.

Step 1 Calculate the output for each pair of input using equation no. 8, 10 & 11.

Step 2 Calculate the error using the following relation Ei = (Oi - Di) (12)

Where, Di is the desired output s[k-d].

Step 3 Calculate the mean square error for convergence as

$$E = 0.5 * \sum Ei^2 / N \tag{13}$$

Where, N is the total number of training patterns and a multiplication factor of 0.5 has been taken to simplify the calculations.

Step 4 Different weights of the networks are updated as following

(a) Weights associated with  $\Sigma 1$  and  $\Sigma 2$  part of the GN are updated as  $W(k) = W(k-1) + \Delta W$  (14) Where,  $\Delta W = \eta \delta_k (Os - Op) + \alpha W(k-1)$ 

and 
$$\delta_k = \sum (Oi - Di)$$

GN are updated as  $Wsi(k) = Wsi(k-1) + \Delta Wsi$ Where,  $\Delta Wsi = \eta \delta sYi + \alpha Wsi(k-1)$ 

(b) Weights associated with inputs and  $\Sigma 1$  part of the

And, 
$$\delta s = \sum \delta_k W (1 - Os) * (1 + Os)$$
 (15)

(c) Weights associated with inputs and  $\Pi$  part of GN are updated as  $Wpi(k) = Wpi(k-1) + \Delta Wpi$ 

Where,  $\Delta Wpi = \eta \delta_n Yi + \alpha Wpi(k-1)$ 

and

$$\delta_p = \sum \delta_k (1 - W) * (-2 * prod \_p) * Op \quad (16)$$

Where,  $\eta$  is the learning rate and  $\alpha$  is the momentum factor, whose values ranges between 0 and 1 determined by trial and error.

# 4. SIMULATION RESULTS AND DISCUSSION

Following channel model is used to simulate the channel.

$$H(z)=0.3482 + 0.8704 z^{-1}+0.3482 z^{-2}$$
 (17)

For linear channel NL=0, 
$$y'[k] = x[k]$$
 (18)

The nonlinear channel, NL=1 is modeled to nonlinearity introduced due to saturation of amplifiers used in the transmission systems as

$$y'[k] = tanh (x[k]) + q[k]$$

$$\frac{X(z)}{S(z)} = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2}$$
 (19)

NL = 2 is modeled to random nonlinear distortions as

$$y'[k] = x[k] + 0.2 x^{2}[k] - 0.1 x^{3}[k] + q[k]$$
 (20)

Fourth order, m=4 equalizer with delay, d=1 is simulated.

Channel equalizer is implemented using GN. For training a random sequence of 1000 duobinary signals of  $\{1, -1\}$ , equi-probable and independent identically distributed (i.i.d.) is generated and passed though the channel. Nonlinearities and white Gaussian noise are further introduced. Initial weights are generated randomly. This generated sequence is used to train the equalizer with back propagation learning algorithm for 300 epochs to obtain minimum mean square error. Values of  $\eta$  and  $\alpha$  are chosen as 0.0015 and 0.5 respectively.

Fig (4) shows the convergence characteristics for the three nonlinear channel models for SNR=16 dB. The characteristics show fast and smooth convergence of error for all the three nonlinear channel models and make it suitable for real time applications.

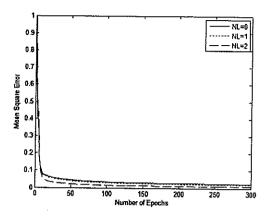


Figure 4: Convergence Characteristic of the Equalizers For The Three Channel Models At SNR=16 DB

The trained equalizer is tested using separately generated duo-binary, equi-probable and i.i.d. sequence with nonlinearities and white Gaussian noise added. The results are averaged over 10 repetitions using testing sample of size 10000 each. SNR is varied between 2-20 db in steps of 2 db to ascertain performance under different noise conditions. Fig (5-7) shows the plot of BER performance of channel for the channel with NL=0, NL=1 and NL=2 respectively which clearly shows the capability of the GN based equalizer to reconstruct the received destroyed signals. The BER performance is also compared with conventional LTE LMS equalizer and optimal Bayesian solution. BER of the proposed equalizer outperforms conventional filter based LMS equalizer and approaches to Bayesian performance even when the equalizer is operated under severe nonlinearity and low SNR conditions. Superior performance of GN based equalizer over linear LMS equalizer for all the three channel models is quite evident from the figures 5-7. There is severe BER performance degradation in the conventional LMS equalizer as the severity of nonlinearity increases while the BER performance of the GN based equalizer is quite similar to each other for both

the linear and nonlinear channel models and approaches to optimal Bayesian solution especially for severe nonlinearity NL=2 through out wide variation of SNR.

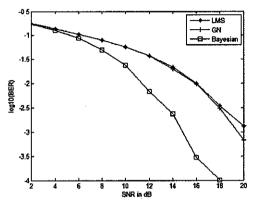


Figure 5 : BER performance of the Channel NL=0

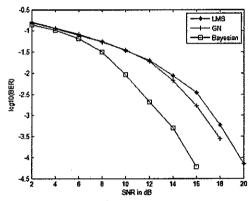


Figure 6 : BER Performance of the Channel
NL=1

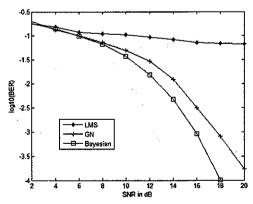


Figure 7: BER Performance of the Channel with NL=2

#### 5. Conclusion

Computationally efficient GN based equalizer is described. There is no problem of selection of initial architecture of neural network as only a single GN is required. No hidden layer is required. Reduced complexity and much simple design procedure is required. Fast convergence of error during training is achieved because it has a much smaller number of weights to be adjusted hence suitable for real time applications. The simulation results show that this neuron provides flexibility and fault tolerant capability to cope up with the nonlinearities involved and that proposed equalizer BER performance outperforms conventional filter based LMS equalizers and approaches to optimal approaches to optimal Bayesian solution for both linear and nonlinear conditions. GN based equalizers offer the advantages of both reduced complexity and good acceptable BER performances hence attractive alternatives for designing equalizers for digital communication systems.

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