# Index Based Internet Araffic Sharing Analysis of Users by a Markov Chain Probability Model

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#### ABSTRACT

The increasing demand of Internet access caused a variety of congestions in the networks, which could be reduced by the better management offered due to service providers. This leads to an assessment of the quality of service (QoS) of operators by the users. This paper takes into account the set-up of two Internet Service Provider (ISP) environments and a Markov chain model is applied over the Internet access behavior of the user indices for different types of users are defined and compared under model approach using simulation study. The blocking probability of the competitor ISP has a significant impact on the initial preference.

**Keywords:** Morkov Chain Model, Initial Preference, Call-by-Call Basis, Internet Service Provider [ISP or operators], Internet Access, Network Congestion, Users Behaviour.

## 1. Introduction

In every walk of life, the use of Internet is being in practice in modern days of electronic communication. More and more people are joining to the club of Internet users in the entire world. This has generated a high

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<sup>2</sup>Dept.of Computer Science and Applications, Dr.H.S.Gour University, Sagar, M.P - 470003. Email:sanjaymca2002@yahoo.com amount of traffic load on the network due to rigorous emergence of call requests every minute. Excess traffic load constitutes immense trouble in the form of congestion in the flow of information through one or more networks. Large numbers of Internet Service providers (ISP) are coming up in the market, making this a business of quality oriented. Consumers need high quality of service for call connectivity and this very ground used to decide about their initial market preference to an ISP. Some users may be loyal and dedicated to an ISP and some may casual in their approach. The pre- set starting choices (or liking) to an ISP could be an evaluation factor for market capture in a competitive market.

This paper presents an analysis of initial preference of an ISP by the different categories of users present in the market. Index for user categories are formulated and compared with each other. Naldi [16] discussed quality of service (QoS) and user behavior under two-operator environment in the setup of four states only. We add a fifth state whose impact has been examined in this content. Deriving further motivation from Naldi [15],[17] and applying theories, techniques of Medhi [12], Perzen [11], Yuan and Lygeros [3], a Markov chain model is used to study the behavior of various groups of consumers in the competitive market situation. Some other useful contributions for the use of Markov chain models are due to [1], [2], [4], [5], [6], [7], [8], [9], [10], [13], [14] and [18]. In order to avoid the mathematical complications, only two ISP are taken into consideration in this contest.

## 2. USER BEHAVIOUR FOR INTERNET ACCESS

Suppose  $ISP_1$  and  $ISP_2$  are two Internet service providers in a market, each having predetermined market capture by a% and b% respectively (a + b = 100%). Assume the followings:-

- (i) A user connects his call through either ISP, or by ISP,.
- (ii) User attempts for an ISP only once and then shift to the next ISP in next call attempt and so on.
- (iii) There are three other options for a user like (a) go to rest (b) abandon the attempt process (c) get success during call attempt.
- (iv) The initial probability of selection to  $ISP_1$  is p = a / 100, and to  $ISP_2$  is (1-p) = b/100.
- (v) The blocking probability in network due to congestion experienced by ISP<sub>1</sub> is L<sub>1</sub> and by ISP<sub>2</sub> is L<sub>2</sub>.

#### 3. Markov Chain Model For User Behaviour

Let  $\{Y^{(n)}, n \ge 0\}$  be a Markov chain having transitions over the state space  $\{ISP_1, ISP_2, RS, SS, AP\}$ , where  $Y^{(n)}$  denotes the position of random variable Y at the  $n^{th}$  call attempt  $(n \ge 0)$ , and five states in state space are:

State ISP,: First Internet service provider.

State ISP,: Second Internet service provider.

State RS: Taking rest for a short duration

State SS : Success obtained in call connection

State AP : Leaving the process of call attempt

Suppose the user is on ISP<sub>1</sub> at  $n^{th}$  attempt. If this call is blocked with the probability L<sub>1</sub> then he shifts either to ISP<sub>2</sub> or to RS state in the  $(n + 1)^{th}$  attempt. User can not

be at the same state in two consecutive call attempts except at SS and AP. User abandons the call attempt process at  $(n + 1)^{th}$  attempt with probability  $p_A$ . If he reaches to RS from  $ISP_2$  in  $n^{th}$  attempts then in next  $(n + 1)^{th}$  he may either be on  $ISP_1$  with probability r, or on  $ISP_2$  with probability (1 - r). From RS user can not move to states SS and AP. The diagrammatic form of state transition mechanism is in fig 3.1 and transition probability matrix is in fig 3.2.

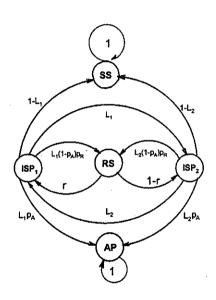


Figure 3.1: Transition Diagram

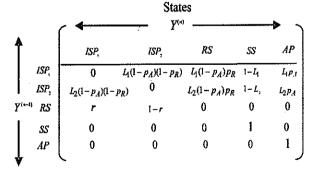


Figure 3.2: Transition Probability Matrix

#### (a) Faithful User (FU)

A user who is dedicated to ISP<sub>1</sub>, otherwise prefer to rest state RS but does not attempt for ISP<sub>2</sub>. The converse of it happens for ISP<sub>2</sub>. A group of this kind of users is defined as faithful users (FU) for ISP<sub>1</sub> {or ISP<sub>2</sub>}.

## (b) Partially Impatient User (PIU)

A user who attempts between ISP<sub>1</sub> and ISP<sub>2</sub> only all the time until call completes or abandon the process but never prefers to reach to RS. A group of this behavior is categorized as PIU.

# (c) Completely Impatient User (CIU)

A user who attempts to  ${\rm ISP_1}$  or  ${\rm ISP_2}$  or RS or abandon in  ${\rm n^{th}}$  attempt. A group of user with such behavior is termed as CIU.

The  $n^{th}$  step transitions probability for FU to  $ISP_{_{1}}$  for  $n\geq 0$  is

$$P\left[Y^{(2n)} = ISP_1\right] = pE^n$$

$$P\left[Y^{(2n+1)} = ISP_1\right] = 0 \qquad --- (1)$$

$$P\left[Y^{(2n)} = ISP_2\right] = (1-p)D^n$$

$$P\left[Y^{(2n+1)} = ISP_2\right] = 0 \qquad --- (2)$$
where  $E = B_1 r$ ,  $D = B_2(1-r)$ ,  $B_1 = L_1(1-p_A)p_R$ ,  $B_2 = L_2(1-p_A)p_R$ ,  $n = 0,1,2,3 ---$ 

For PIU, the nth step transition probability is :-

$$P\left[Y^{(2n)} = ISP_1\right] = pC^n$$

$$P\left[Y^{(2n+1)} = ISP_1\right] = (1-p)A_2C^n$$

$$P\left[Y^{(2n)} = ISP_2\right] = (1-p)C^n$$

$$P\left[Y^{(2n+1)} = ISP_2\right] = pA_1C^n$$

where 
$$C=A_1\,A_2$$
 ,  $A_1=L_1(1-p_A)$   $(1-p_R)$ ,  $A_2=L_2(1-p_A)$   $(1-p_R)$ 

For Completely Impatient User (CIU), the  $n^{th}$  attempt approximate expressions of probability are:

$$P[Y^{(2n)} = ISP_1] = p(C+E)^n$$

$$P[Y^{(2n+1)} = ISP_1] = (1-p)A_2(C+D+E)^n$$

$$P[Y^{(2n)} = ISP_2] = (1-p)(C+D)^n$$

$$P[Y^{(2n+1)} = ISP_2] = pA_1(C+D+E)^n$$

## 4. USER INDEX

We define user indices for all the three user categories FU, PIU, CIU for operator ISP, as:

## (A) For Faithful Users (FU)

$$[I^{(n)}(ISP_1^n)]_{FU} = \frac{P(Y^{(n)} = ISP_1^n)_{FU}}{\left[P(Y^{(n)} = ISP_1^n)_{FU} + P(Y^{(n)} = ISP_1^n)_{PIU} + P(Y^{(n)} = ISP_1^n)_{CIU}\right]}$$

# (B) For Partially Impatient Users (PIU)

$$[I^{(n)}(ISP_{i}^{n})]_{PIU} = \frac{P_{i}Y^{(n)} = ISP_{i}Y_{i}U_{i}}{[P_{i}Y^{(n)} = ISP_{i}Y_{i}U_{i} + P_{i}Y_{i}U_{i} = ISP_{i}Y_{i}U_{i} + P_{i}Y_{i}U_{i} = ISP_{i}Y_{i}U_{i}]}$$

# (C) For Completely Impatient Users (CIU)

$$[I^{(n)}(ISP_{i})]_{CIU} = \frac{P_{i}Y^{(n)} = ISP_{i}|_{CIU}}{\left[P_{i}Y^{(n)} = ISP_{i}|_{FU} + P_{i}Y^{(n)} = ISP_{i}|_{DIU} + P_{i}Y^{(n)} = ISP_{i}|_{CIU}\right]}$$

Similar expressions of user indices could be written for operator  $ISP_2$ . These indices are function of four parameters  $L_{i_1}L_{i_2}p_{i_1}$  and  $i_2$ 

### 5. SIMULATION BASED INDEX ANALYSIS

Indices are examined over number of increasing attempts as function of blocking probabilities.

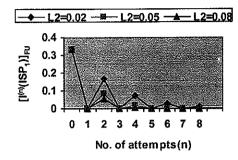


Figure 5.1 :  $L_1 = 0.3, p_A = 0.05, p_R = 0.03, r = 0.3$ 

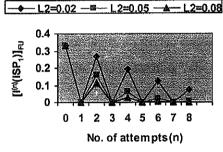


Figure 5.2 :  $L_1 = 0.3, p_A = 0.05, p_R = 0.03, r = 0.7$ 

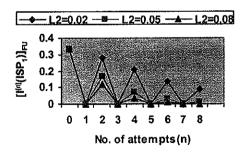


Figure 5.3 :  $L_1 = 0.3, p_A = 0.05, p_R = 0.07, r = 0.3$ 

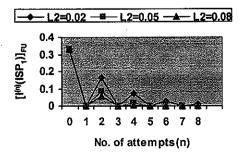


Figure 5.4 :  $L_1 = 0.7, p_A = 0.05, p_R = 0.03, r = 0.3$ 

From fig 5.1 - 5.4 (Index of FU for ISP<sub>1</sub> over varying L<sub>2</sub>), it is evident that with the increase of blocking probability of opponent, the operator ISP<sub>1</sub> looses index value for faithful users. But, however, with the increase of rest state probability r, this index has growing trend. So the rest state plays an important role in the improvement of FU. The increase in  $p_R$  value also have a positive impact on growth of index value.

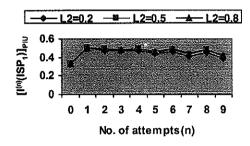


Figure 5.5 :  $L_1 = 0.3, p_A = 0.05, p_R = 0.03, r = 0.3$ 

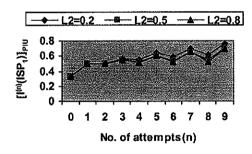


Figure 5.6 :  $L_1 = 0.3, p_A = 0.05, p_R = 0.07, r = 0.3$ 

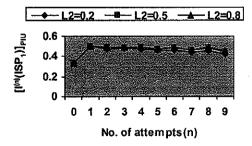


Figure 5.7 :  $L_1 = 0.7$ ,  $p_A = 0.05$ ,  $p_R = 0.03$ , r = 0.3

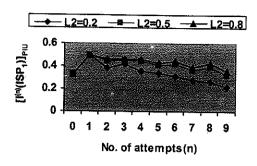


Figure 5.8 :  $L_1 = 0.3, p_A = 0.05, p_R = 0.07, r = 0.7$ 

Looking over to fig. 5.5 - 5.8 (Index of PIU for ISP<sub>1</sub> over varying  $L_2$ ), the opponent's blocking probability  $L_2$  also has an impact on the group strength of partially impatient users of ISP<sub>1</sub>. The index for PIU decreases with increasing r whereas simply with high  $L_2$ , the index shows almost a stable pattern over increasing number of attempts.

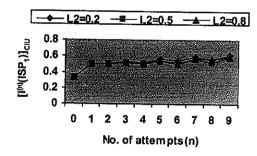


Figure 5.9 :  $L_1 = 0.3, p_A = 0.05, p_R = 0.03, r = 0.3$ 

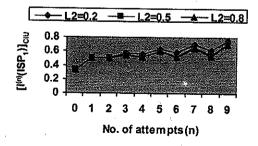


Figure 5.10 :  $L_i = 0.3, p_A = 0.05, p_R = 0.07, r = 0.3$ 

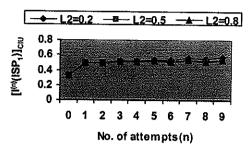


Figure 5.11 :  $L_1 = 0.7$ ,  $p_A = 0.05$ ,  $p_R = 0.03$ , r = 0.3

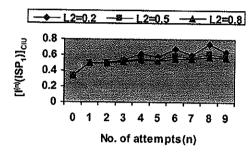


Figure 5.12 :  $L_1 = 0.3$ ,  $p_A = 0.05$ ,  $p_R = 0.07$ , r = 0.7

On comparing index of completely impatient user (CIU), (from fig. 5.9-5.12) with increasing  $L_2$ , the probability index for CIU increases. When r is high, the index increases for the lower values of L2. Variation of pR has similar effect as that of variation of r .The CIU index gets a stable value over larger number of attempts with small L1, L2, r and p<sub>R</sub>. Overall views indicate the probability index for FU has a constant fluctuating nature over large number of attempts. Contrary to this, index of PIU has little fluctuation preferably in the range (0.3 -0.5). Moreover stability pattern occurs in this range too over large number of attempts. The smaller choice of parameter is preferable and advisable. The index variation of PIU and CIU are very much similar than FU but the stability pattern over large attempt is shifted to higher value (0.4 - 0.6) than the PIU.

#### 6. Conclusion

With the hike of blocking chance of opponent, the operator ISP, looses index of faithful users but with little increment in rest state probability this index increases. The rest state has an important role in the development of major stuff of faithful users. Actual benefit starts when the proportion of faithful users is high for an operator. Opponent's blocking probability L, does not affect much to the index of partially impatient users over increasing number of attempts. However, the rest state probability also affects this index. Coming over to the third category, the increase in L, has a positive impact to the index increment for completely impatient users. Overall, this is to observe that blocking probability level has a vital role in causing the variations in user indices. The rest state probability also affects these indices. Therefore, it is recommended to open up a restaurant outside to an Internet shop to act like a rest state which would contribute much in attracting Internet traffic.

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