A Novel Delta-Rule Approach for the Design of PID Controllers to Stabilize Linear Time Invariant Continuous Systems

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ABSTRACT

This paper presents a systematic delta-rule approach for tuning the parameters of a PID controller during design phase. A suitable second order approximant of the given higher order Linear Time Invariant Continuous system is considered for determining a possible set of the Proportional (K_p), Integral (K_i) and Derivative (K_d) factors of the required PID controller. For stabilization, Overshoot, Settling Time and Steady state error are taken as the design criterion in the Time domain to adjust the values of K_p, K_i and K_d. The controller designed for the Second order approximant is cascaded with the given higher order system to validate the stabilization process. The proposed design methodology is illustrated with transfer functions of selected plants taken from the literature.

KEYWORDS: PID control, Process control, Linear systems, Model Reduction, Second order systems

1. INTRODUCTION

PID-controllers play a dominant role in Process control. These are widely used in Industrial control systems because of the reduced number of parameters to be tuned. Industrial application of PID controllers demands simple and transparent design procedures. The most employed PID design technique used in the industry is the Ziegler–Nichols method [1], which avoids the need for a model of the plant to be controlled and relies solely on the step response of the plant. The main restriction of Ziegler–Nichols method is that it is suitable only for systems with monotonic step response (S-shape response). Hang *et al* [2] have reexamined the Ziegler–Nichols method and proposed new tuning formulae and introduced a setting-point weight for systems with PID controllers. Zhuang and Atherton [3] proposed an optimal design of PID controllers based on the minimization of an integral criterion [integral of the square of the product of time and error (ISTE)]. Yeung et al., [4] presented graphical design method for common continuous-time and discrete-time compensators. The method is based upon a set of Bode design charts, which have been generated using appropriately normalized compensator transfer functions. Several methods for designing controllers have been developed by employing frequency response matching technique. The method proposed by Rattan et al., [5] is based on complex curve fitting technique and involves the matching of frequency response of closed-loop system with that of a reference model. The complex curve-fitting method of Rattan does not guarantee a stable controller. In the method proposed by Houpis [6], the sampled-data system is approximated by a pseudo-continuous-time control system. The approach is applicable to systems with sampling time much smaller than one second. The digital controller design method proposed by Inooka et al., [7] is based on series expansion of pulse transfer function. Aguirre [8] proposed a method for the design of continuous time controllers by matching a com-

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bination of time-moments and Markov parameters of the closed-loop system. The main purpose of all the approaches is to reduce the excessive overshoot of systems to be compensated. Each method has their applications and limitations.

The design of PID controllers can be carried out either in the Time or Frequency domain [9]. The choice of the design domain depends upon the preference of the designer. In most cases, time domain specifications such as Overshoot, Settling time and Steady state error are used to measure the overall system performance. Also, to an inexperienced designer, it is difficult to comprehend the physical connection between frequency domain specifications such as gain and phase margin to actual performance. To ensure a broad acceptance for the controller design, it should be usable by designers with moderate theoretical background and should be applicable for a variety of plants.

In this paper a systematic approach based on a set of proposed delta-rule is presented for tuning the parameters of a PID controller for design purpose. Designing a PID controller for a given higher order systems is computationally intensive and cumbersome. For reducing the computational burden and also to establish an iterative procedure for updating the controller parameter values, a second order approximant of the given higher order system or plant is considered for design purpose. When the PID controller is found to be stabilizing the second order system within the given design specifications, it is cascaded with the original higher order system to validate the actual stabilization process. Unit step response of the closed loop system with the controller and the plant is considered to validate the design.

The rest of the paper is organized as follows. Section 2 defines the problem to be solved. Section 3 outlines the proposed design methodology and also sketches the algorithm for tuning the controller parameters. Numerical illustrations are presented in Section 4, followed by a short discussion.

2. Problem Definition

The purpose of designing a PID controller is to provide control signals that are proportional to:

(i) the error between the reference signal and the actual output (proportional action),

(ii) the integral of the error (integral action), and

(iii) the derivative of the error(derivative action)

This can be mathematically represented as [10],

$$u(t) = K_p\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{d}{dt}e(t)\right]$$

(2.1)

where $u(t)$ and $e(t)$ denote the control and the error signals, respectively, and , K_p, T_i and T_d are the parameters to be tuned. The corresponding transfer function is given as

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \qquad (2.2)$$

Equation (2.2) can be rewritten as,

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \qquad (2.3)$$

The main features of PID controllers are:

(i) the capacity to eliminate steady-state error of the response to a step reference signal (because of integral action), and

(ii) the ability to anticipate output changes (when derivative action is employed).

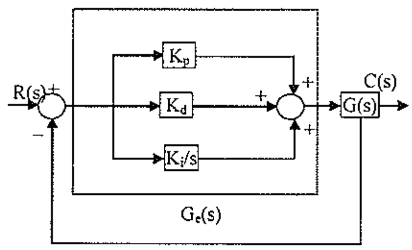

Fig 2.1 Block diagram of a PID controller

The standard block diagram of PID controller is shown in fig 2.1. The proportional control action multiplies the error signal with a constant to improve the overall gain of the system. In fig 2.1, $G(s)$ is the open loop transfer function of the higher order system or plant to be stabilized. The closed loop transfer function of the unity feedback system with can be represented as,

$$T(s) = \frac{G(s)}{1+G(s)} \qquad (2.4)$$

If the output response of $T(s)$ is not stable within the specified design specifications, then the PID controller is cascaded to the forward path to adjust the response. The corresponding transfer function of the unity feedback system can be represented as,

$$T_c(s) = \frac{G(s)G_c(s)}{1+G(s)G_c(s)} \qquad (2.5)$$

The problem is to identify a set of (K_p, K_i, K_d) such that the unit step time response of $T_c(s)$ yields a stable output within certain design specifications. The design specifications considered are Overshoot, Settling time and the Steady state error limits. The limiting values of these parameters depend upon the type of the plant being controlled. For illustration purpose, the following specifications are used in this paper:

(i) Overshoot d" 1%

(ii) Settling time d" 0.5 secs

(iii) Steady state error d" 1%

3. PID Controller Design

3.1 Proposed Methodology

1. Let $G(s)$ be the general form of the transfer function of an n^{th} order Linear Time Invariant Continuous system represented as

$$G(s) = \frac{a_n s^{n-1} + a_{n-1} s^{n-2} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0} \qquad (3.1)$$

2. Let $G_2(s)$ be a Second order approximant of $G(s)$ in the form,

$$G_2(s) = \frac{As+B}{s^2 + Cs + D} \qquad (3.2)$$

3. Let K_p, K_i and K_d respectively be the values of the Proportional, Integral and Derivative factors of the required PID controller.

4. The transfer function of the required PID controller can be written as:

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s$$
$$= \frac{K_d s^2 + K_p s + K_i}{s} \qquad (3.3)$$

5. Applying Pole-zero cancellation technique [9], from equations (3.2) and (3.3) we can have

$$K_d s^2 + K_p s + K_i = s^2 + Cs + D \qquad (3.4)$$

6. From (2.4), the initial value of can be taken as . Also, the following approximate relations can be established between , and .

$$\frac{K_p}{K_d} = C \qquad (3.5)$$

$$\frac{K_i}{K_d} = D \qquad (3.6)$$

7. Taking partial derivatives of (3.5) and (3.6), we get the following delta rules for updating the values of K_p, K_i and K_d:

$$\Delta K_p = C(\Delta K_d) \qquad (3.7)$$

$$\Delta K_i = D(\Delta K_d) \qquad (3.8)$$

8. Let the initial values of (K_p, K_i, K_d) be $(C, 0, 0)$ respectively.

9. Let the design specifications or figure of merits be specified in terms of the %Overshoot (O_{max}),

Settling Time (T_s) and the Steady state error (SS_{err}) values.

10. Form the transfer function of the closed loop system of $G_c(s)$ and $G_2(s)$ with unity feedback as:

$$T_c(s) = \frac{G_c(s)G_2(s)}{1+G_c(s)G_2(s)} \quad (3.9)$$

11. From the unit step time response of $T_c(S)$, record $O_{\max}, T_s, SS_{err}$.

12. If the design specifications are met, then declare the values of K_p, K_i, K_d and further tuning is not required.

13. If the design specifications are not met, then tune as follows:

14. Till the is within the design specifications, do the following:

    (i) Increment K_p by a small value $\Delta K_p = 0.5$

    (ii) Using the relation in equation (3.7) find the increment in K_d as $\Delta K_d = \frac{\Delta K_p}{C}$

    (iii) Reformulate $G_c(s)$ as in equation (3.3)

    (iv) Reformulate $T_c(s)$ as in equation (3.9)

    (v) From the unit step time response of , record .

15. Till T_s and SS_{err} are within the design specifications, do the following:

    (i) Increment K_i by a small value $\Delta K_i = 0.5$

    (ii) Using the relation in equation (3.8) find the increment in K_d as $\Delta K_d = \frac{\Delta K_i}{D}$

    (iii) Reformulate $G_c(s)$ as in equation (3.3)

    (iv) Reformulate $T_c(s)$ as in equation (3.9)

    (v) From the unit step time response of , record and

16. Declare the values of K_p, K_i, K_d.

17. Form the transfer function of the closed loop system of $G_c(s)$ and $G\ (s)$ with unity feedback as:

$$T_c'(s) = \frac{G_c(s)G\ (s)}{1+G_c(s)G\ (s)} \quad (3.10)$$

18. Observe the unit step time response of $T(s)$ to verify the expected stabilization effect of the PID controller $G_c(s)$ on the given system $G(s)$ represented by equation (3.1).

The algorithm for tuning the values of K_p, K_i, K_d is given in the following section.

1.2 Algorithm for Tuning K_p, K_i, K_d

Input:

1. Transfer function of Second order system :

   a. Vector of the coefficients of numerator:

      nr ← [*0 A B*]

   b. Vector of the coefficients s of denominator:

      dr ← [*1 C D*]

2. Figure of Merits / Design Specifications.

a. Overshoot : *spec_overshoot*

b. Settling time: *spec_tsettling*

c. Steady state error: *spec_ss*

Output:

1. Computed values of *kp, ki, kd*, where *kp, ki* and *kd* denotes the proportional, integral and derivative control factors .

Tune_Kp_Ki_Kd (nr, dr, spec_overshoot, spec_tsettling, spec_ss)

1. [initialization]

   kp ← *c; kd* ← *0; ki* ← *0*

2. [Initialization for adjusting overshoot to be within specified range]

   overshoot ← ***calc_overshoot (kp,ki,kd,nr,dr)***

   delta ← *0*

3. while *overshoot is not within (1 + spec_overshoot)* do:

delta ← *0.5*

kp ← *kp + delta*

kd ← *kd + delta / C*

overshoot ***calc_overshoot(kp,ki,kd,nr,dr)***

4. end of while
5. [Initialization for adjusting settling time and steady state error to be within range]

delta ← *0*

settling_time ← ***calc_settlingtime(kp,ki,kd,nr,dr)***

steady_state_error

← ***calc_steady_state_error(kp,ki,kd,nr,dr)***

6. while (*settling time not < spec_tsettling)* ***or*** *(steady state not < spec_ss)* do:

delta ← *0.5*

ki ← *ki + delta*

kd ← *kd + delta / D*

settling_time ← ***calc_settlingtime(kp,ki,kd,nr,dr)***

steady_state_error

← ***calc_steady_state_error(kp,ki,kd,nr,dr)***

7. end of while
8. print values of *kp*, *ki* and *kd*.
9. End

calc_overshoot(kp,ki,kd,nr,dr)

1. Form the transfer function of the PID controller with *kp ,ki ,kd*
2. Form the closed loop transfer function of the Second Order system attached to the PID controller
3. Plot the unit step time response of the closed loop system
4. *overshoot* ← *Percentage overshoot of the closed loop system*
5. *return (overshoot)*

end

calc_settlingtime(kp,ki,kd,nr,dr)

1. Form the transfer function of the PID controller with *kp ,ki ,kd*
2. Form the closed loop transfer function of the Second Order system attached to the PID controller
3. Plot the unit step time response of the closed loop system
4. *settling_time* ← *Settling time of the closed loop system*
5. *return (settling_time)*

end

calc_steady_state_error(kp,ki,kd,nr,dr)

1. Form the transfer function of the PID controller with *kp ,ki ,kd*
2. Form the closed loop transfer function of the Second Order system attached to the PID controller
3. Plot the unit step time response of the closed loop system
4. *steady_state_error* ← *Steady state error of the closed loop system*
5. *return (steady_state_error)*

4. Numerical Illustrations

Illustration 1.

1. Consider the transfer function of the Eighth order Linear Time Invariant continuous system from [11] represented as

$$G(s) = \frac{\begin{pmatrix}19.82s^7 + 429.26156s^6 + 4843.8098s^5 \\ + 45575.892s^4 + 241544.75s^3 + \\ 905812.05s^2 + 1890443.1s + 842597.05\end{pmatrix}}{\begin{pmatrix}s^8 + 30.41s^7 + 358.4295s^6 + 2913.8638s^5 \\ + 18110.567s^4 + 67556.983s^3 + \\ 173383.58s^2 + 149172.19s + 37752.826\end{pmatrix}} \quad (4.1)$$

2. Applying the Rule-Based approach [Appendix I], the Second order system formulated is:

$$G_2(s) = \frac{19.82s + 2.1864}{s^2 + 0.5634s + 0.0980} \quad (4.2)$$

3. Initial values of $(K_p, K_i, K_d) = (0.5634, 0, 0)$
4. The proposed Tuning algorithm is invoked with the following parameters:
Overshoot $\leq$ 1%
Settling Time $\leq$ 0.5 secs
Steady state error 1%
5. The values of the parameters returned by the algorithm are:

$$K_p = 3.0634,\ K_i = 0.5000,\ K_d = 9.5393$$

6. The Transfer function of the designed PID controller is:

$$G_c(s) = \frac{K_d s^2 + K_p s + K_i}{s}$$

$$= \frac{9.5393s^2 + 3.0634s + 0.5000}{s} \quad (4.3)$$

7. The closed loop transfer function of the PID controller represented by $G_c(s)$ in equation (4.3) attached to the Second order system represented by $G_2(s)$ in equation (4.2) is obtained as:

$$T_c(s) = \frac{G_c(s)G_2(s)}{1 + G_c(s)G_2(s)} \quad (4.4)$$

8. The closed loop transfer function of the PID controller represented by $G_c(s)$ in equation (4.3) attached to the Eighth order system represented by $G(s)$ in equation (4.1) is obtained as:

$$T_c'(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} \quad (4.5)$$

9. The unit step time responses $G(s)$, $T_c(s)$ and $T_c'(s)$ represented by equations (4.1), (4.4) and (4.5) are shown in Figure 4.1(a), Figure 4.1(b) and Figure 4.1(c) respectively.

10. From Figure 4.1(c), it is note that designed PID controller is suited for the given over damped higher order system yielding favorable unit step time response subject to the given design constraints.

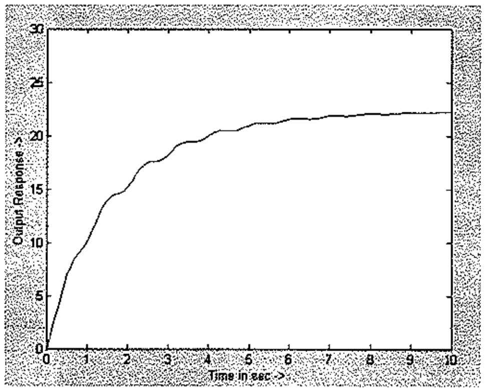

Figure 4.1(a) Unit Step Time response of the Eighth Order System for Illustration 1

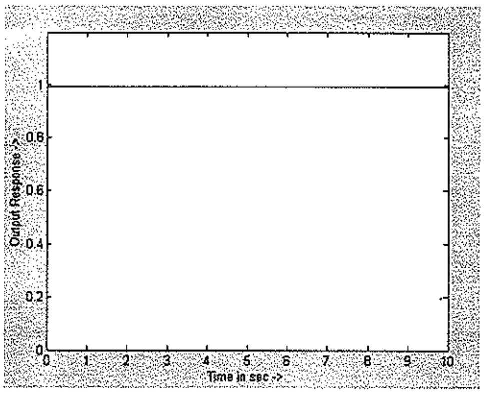

Figure 4.1(b) Unit Step Time Response of the Second Order System with the PID controller for Illustration 1

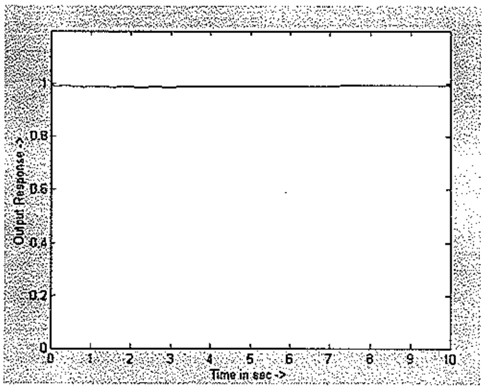

Figure 4.1(c) Unit Step Time Response of the Eighth Order System with the PID controller for Illustration 1

Illustration 2.

1. Consider the transfer function of the Eighth order Linear Time Invariant continuous system from [12] represented as

$$G(s) = \frac{\begin{pmatrix} 35s^7 + 1086s^6 + 13285s^5 \\ + 82402s^4 + 278376s^3 + \\ 511812s^2 + 482964s + 1994480 \end{pmatrix}}{\begin{pmatrix} s^8 + 21s^7 + 220s^6 + 1558s^5 \\ + 7669s^4 + 24469s^3 + \\ 46350s^2 + 45952s + 17760 \end{pmatrix}} \quad (4.6)$$

2. Applying the Rule-Based approach [Appendix I], the Second order system formulated is:

$$G_2(s) = \frac{35s + 438.8719}{s^2 + 1.8988s + 40.078} \quad (4.7)$$

3. Initial values of $(K_p, K_i, K_d) = (1.8988, 0, 0)$
4. The proposed Tuning algorithm is invoked with the following parameters:

Overshoot	$\leq$	1%
Settling Time	$\leq$	0.5 secs
Steady state error	$\leq$	1%

5. The values of the parameters returned by the algorithm are:

$K_p = 1.8988$, $K_i = 11.0000$, $K_d = 0.2744$

6. The Transfer function of the designed PID controller is:

$$G_c(s) = \frac{K_d s^2 + K_p s + K_i}{s} = \frac{0.2744s^2 + 1.8988s + 11.0000}{s} \quad (4.8)$$

7. The closed loop transfer function of the PID controller represented by $G_c(s)$ in equation (4.8) attached to the Second order system represented by $G_2(s)$ in equation (4.7) is obtained as:

$$T_c(s) = \frac{G_c(s)G_2(s)}{1 + G_c(s)G_2(s)} \quad (4.9)$$

8. The closed loop transfer function of the PID controller represented by $G_c(s)$ in equation (4.8) attached to the Eighth order system represented by $G(s)$ in equation (4.6) is obtained as:

$$T_c'(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} \quad (4.10)$$

9. The unit step time responses $G(s)$, $T_c(s)$ and $T_c'(s)$ represented by equations (4.6), (4.9) and (4.10) are shown in Figure 4.2(a), Figure 4.2(b) and Figure 4.2(c) respectively.
10. From Figures 4.2(a) and 4.2(c), it can be observed that designed PID controller removes the oscillations of the given Eighth order system and stabilizes it within the design specifications.

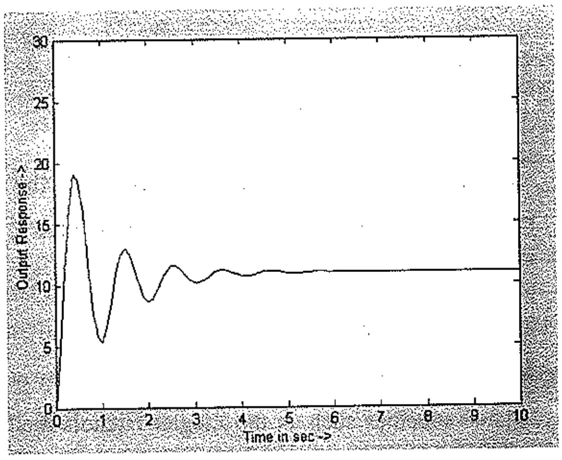

Figure 4.2(a) Unit Step Time response of the Eighth Order System for Illustration 2

5. Discussion

In this paper a new methodology for tuning the parameters of a PID controller has been presented.. The proposed method makes use of a good Second order approximant of the given higher order system to begin the design exercise. The design process is carried out in the

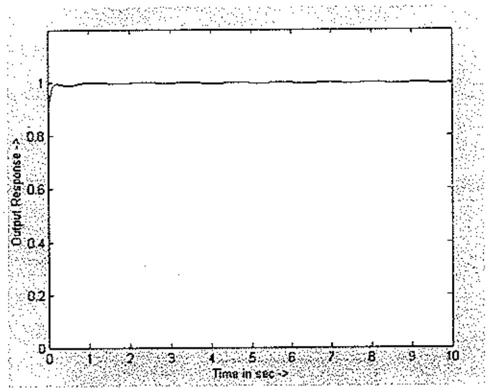

Figure 4.2(b) Unit Step Time Response of the Second Order System with the PID controller for Illustration 2

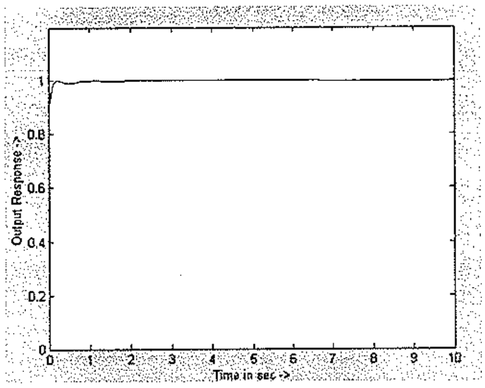

Figure 4.2(c) Unit Step Time Response of the Eighth Order System with the PID controller for Illustration 2

domain with Overshoot., Settling time and the Steady state error as the performance specifications. Unlike the other available schemes such as the Ziegler-Nichols method etc.,[1-8], simple schemes have been proposed for taking the initial values of the control parameters and updating them systematically during the tuning process. As the analysis is done in the time domain, even inexperienced designers can adopt our methodology. The controller designed for the second order approximant is cascaded to the original higher order system and the closed loop system is studied for stability. The methodology is well illustrated with examples taken from the literature and it is found to be straightforward and computationally less intensive. Computer programs have been developed to implement the methodology to assist the designer. The method can also be extended for Discrete and Multivariable system stabilization. For obtaining the second order approximant, a Rule-based approach [Appendix I] proposed by the authors has been used though any other model reduction technique[11-14] can be employed.

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Appendix - I

Rule-based approach for the formulation of Second Order Approximant

Analysis of the original higher order system

1. Let $G(s)$ be the general form of the transfer function of an n[th] order linear time invariant continuous system represented as

$$G(s) = \frac{a_n s^{n-1} + a_{n-1} s^{n-2} + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0} \quad \text{(a.1)}$$

2. The Transient and Steady state gains of this system can be determined using the formulae,

   Transient gain $T_g = a_n / b_n$ (a.2)

   Steady state gain $S_g = a_0 / b_0$ (a.3)

3. *The unit step time response of $G(s)$ is analyzed with a computer program and the Rise Time T_r, Peak Time T_p, Peak Amplitude P_a and Settling Time T_s are noted.*

4. *Observing the unit step time response, the nature of the system represented by $G(s)$ can be classified into one of the categories viz., Aperiodic, Almost Aperiodic or Periodic*

Formulation of initial second order system

5. The general form of the transfer function of a second order system in the s-domain can be represented as,

$$G_2(s) = \frac{As + B}{s^2 + 2\zeta\omega_n + \omega_n^2} \quad \text{(a.4)}$$

   where ζ is the damping ratio and ω_n is the undamped natural frequency of oscillation.

2. The following guideline is used to determine the initial values of ζ and ω_n:

If the nature of the original higher order system represented by the transfer function (a.1) is:

(i) Aperiodic or Almost aperiodic, ζ is taken as 0.9 and ω_n is calculated using the formula, $\omega_n = 4/(\zeta * T_a)$.

(ii) Periodic, ζ is taken as 0.4 and ω_n is calculated using the formula,

$\omega_n = \omega_d (\sqrt{(1-\zeta^2)})$

where, ω_d is the damped frequency of oscillation given by

$\omega_d = 2\pi n / T_s$,

'n' being number of oscillations before the system output settles.

7. Now, the values of A and B corresponding to equation (a.4) can be computed as

$$A = T_g \text{ and } B = S_g / \omega_n^2 \qquad \text{(a.5)}$$

8. The unit step time response of the initial second order system $G_2(s)$ is analyzed with a computer program and its characteristics are noted as listed in step 3.

9. The cumulative error index J_0 using the Integral square error of the unit step time responses of the given higher order system $G(s)$ represented by equation (a.1) and the initial second order system $G_2(s)$ represented by equation (a.4) is calculated.

Rule based approach for Error minimization

10. The values of ζ and ω_n are kept in the working memory.

11. The following rules are considered for updating the current values of ζ and ω_n iteratively.

Rule 1: The allowable range of ζ is 0.9 < < 1 for Aperiodic systems.

Rule 2: When ζ is constant, ω_n is inversely proportional to the Settling time T_s

Rule 3: *When ζ is constant, ω_n is inversely proportional to the Peak time T_p*

Rule 4: *When ζ is constant, ω_n is inversely proportional to the Rise time T_r*

Rule 5: *When ω_n is constant, ζ is inversely proportional to the Settling time T_s*

Rule 6: When ω_n is constant, ζ is inversely proportional to the Peak time T_p

Rule 7: When ω_n is constant, ζ is directly proportional to the Rise time T_r

Rule 8: When ω_n is constant, ζ is inversely proportional to the Peak amplitude P_a

Rule 9: When ζ is constant, ω_n has no influence on Peak amplitude P_a

12. At the end of each iteration, the current values of ζ and ω_n are used to reformulate the transfer function of the second order model, maintaining the Transient and Steady state gains of the original higher order system.

13. The unit step time response of the current second order system is analyzed with a computer program and its characteristics are noted as listed in step 3.

14. The current cumulative error index J_c using the Integral square error of the unit step time responses of the given higher order system $G(s)$ represented by equation (a.1) and the current second order model is calculated.

15. When J_c is greater than the cumulative error index of the previous iteration or J_0, the rule based algorithm stops.

16. The second order system corresponding to the minimum cumulative error index is declared as the winner.

Author's Biography :

Dr. S. N. Sivanandam is Professor and Head of Department of Computer science and Engineering with 35 years of experience in PSG College of Technology. He has won the Who's who award and Best Teacher award. He has published papers in National /International Journals and Conferences and authored books many books. He has guided 15 PhD candidates. He is reviewer of International Journals and has chaired National and International Conferences.