Model Order Reduction using Polynomial Derivative and Genetic Algorithm

C.B. Vishwakarma[1] and R. Prasad[2]

Abstract

The authors present a method for reducing the order of the large-scale linear dynamic single-input-single-output (SISO) systems. The denominator polynomial of the reduced model is determined by using polynomial derivative while the numerator coefficients are computed by minimizing the integral square error between step responses of the original system and reduced order model using Genetic Algorithm. The proposed method is conceptually simple, computer oriented and comparable in quality. This method also guarantees the stability of the reduced model if the original high-order system is stable. The viability of the proposed method is illustrated with the help of numerical examples from the literature.

Keywords: Polynomial derivative, Genetic Algorithm, Order reduction, Integral square error, Stability, Transfer function.

1. Introduction

The analysis of the physical system starts by the building up of a mathematical model which is a representation of an object or system. The approximation of linear systems play an important role in many engineering applications, especially in control system design, where the engineer is faced with controlling a physical system for which an analytical model is represented as a high order linear system. In many practical situations, a fairly complex and high order system is not only tedious but also not cost effective for online implementation. It is therefore desirable that a high-order system be replaced by a low-order system such that it retains the main qualitative properties of the original system. A wide variety of model order reduction methods [1-6] have been proposed by the several authors in frequency domain. Many mixed method taking advantages of two order reduction methods are suggested by the authors [7-10]. Some order reduction methods [11-14] have also been proposed by using error minimization techniques.

The differentiation method or polynomial derivative was first introduced by Gutman et. al [15]. In this method, reciprocal of the numerator and denominator polynomials of the high-order transfer function are differentiated successively many times to yields the coefficients of the reduced order transfer function. The successive differentiation of the polynomial having negative roots maintain the negativeness of the roots of the reduced polynomial but approximates the high modulus roots of the high-order polynomial while the successive differentiation of the reciprocal polynomial also maintain the negativeness but approximates the low modulus roots. This method is also applicable to unstable and non-minimum phase systems. The draw back of this method is that steady-state does not match always. Pal and Prasad [16] combined this method with continued fraction approach and also extended it for multivariable systems.

[1]Lecturer in Faculty of Engineering and Technology, Gurukul Kangri University, Haridwar, India Email : cbv_gkv@yahoo.co.in

[2]Associate Professor in the Department of Electrical Engineering at Indian Institute of Technology Roorkee, India.

In the present study, the authors used this method to get only the denominator polynomial for the reduced order model.

Nowadays, the Genetic Algorithm (GA) has become a popular optimization technique in the field of science and engineering. This is based on the process of Darwin's theory of Evolution. By starting with a set of potential solutions and changing them during several iterations the Genetic Algorithm hopes to converge on the most 'fit' solution. The process begins with a set of potential solutions or chromosomes (usually in the form of bit strings) that are randomly generated or selected. The entire set of these chromosomes comprises a population. The chromosomes evolve during several iterations or generations. New generations (offspring) are generated using the crossover and mutation technique. Crossover involves splitting two chromosomes and then combining one half of each chromosome with the other pair. Mutation involves flipping a single bit of a chromosome. The chromosomes are then evaluated using a certain fitness criteria and the best ones are kept while the others are discarded. This process repeats until one chromosome has the best fitness and thus is taken as the best solution of the problem [17].

In this paper, the denominator polynomial for the reduced order model is determined by using polynomial derivative [15] and the coefficients of the numerator are computed by minimizing the integral square error (ISE) between the step responses of the original system and the reduced order model using GA.

2. Statement of Problem

Let, the transfer function of the high-order original system of the order 'n' be

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + \dots\dots\dots + a_{n-1}s^{n-1}}{b_0 + b_1 s + \dots\dots\dots + b_n s^n} \quad (1)$$

Where $a_i; 0 \le i \le n-1$ and $b_i; 0 \le i \le n$ are known scalar constants. Let, the transfer function of the reduced model of the order 'k' be

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{c_0 + c_1 s + \dots\dots\dots + c_{k-1}s^{k-1}}{d_0 + d_1 s + \dots\dots\dots + d_k s^k} \quad (2)$$

Where $c_i; 0 \le i \le k-1$ and $d_i; 0 \le i \le k$ are unknown scalar constants.

The objective of this paper is to find the reduced order model in the form of equation (2) from the original system (1) such that it retains the qualitative properties of the original high-order system.

3. Reduction Procedure

The order reduction procedure consists of the following two steps:

Step-1: Determination of the denominator polynomial for the k^{th}-order reduced model using polynomial derivative [15]:

First, the denominator polynomial $D(s)$ of the original system $G_n(s)$ is reciprocated as

$$\overset{*}{D}(s) = s^n D\left(\frac{1}{s}\right) \quad (3)$$

Now the reciprocated polynomial $\overset{*}{D}(s)$ is differentiated $(n-k)$ times and then resulting polynomial is again reciprocated back using the equation (3), which gives the k^{th}-order denominator polynomial as

$$D_k(s) = d_0 + d_1 s + \dots\dots\dots + d_k s^k \quad (4)$$

The coefficients of $D_k(s)$ can also be evaluated from the recursive relation [18], which is given as

$$d_{i-n+k} = b_{n-i}[i!/(i-n+k)]s^{i-n+k} \quad (5)$$

for $i = (n-k)$ to n

Step-2: Determination of the coefficients of the numerator using GA:

For finding the numerator coefficients, the Genetic Algorithm is applied to minimize the objective function 'J' [1] which is an integral square error in between the time responses of the original system and the reduced order model and is given by

$$J = \int_0^{\infty} [y(t) - y_k(t)]^2 dt \qquad (6)$$

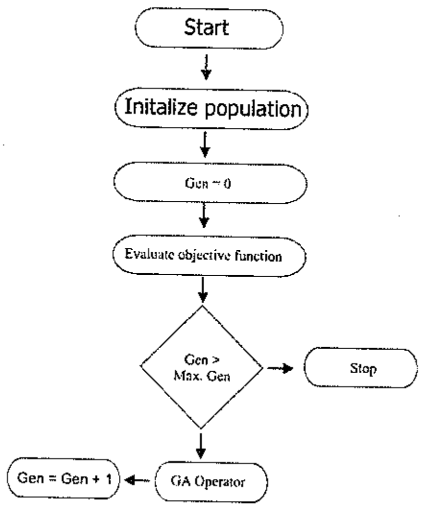
Where $y(t)$ and $y_k(t)$ are unit step responses of the original and reduced system respectively.

The objective function 'J' is the function of the numerator coefficients $c_i; 1 \le i \le k-1$ only, while the numerator coefficient C_0 can be obtained from equation (7) in order to match the steady-state values of the original and reduced system.

$$c_0 = \frac{a_0}{b_0}.d_0 \qquad (7)$$

The suitable GA parameters must be taken which may be different for the different problems.

The flowchart of the Genetic Algorithm is shown in the Fig.1



Start Start

Figure 1: Flowchart of Genetic Algorithm

Examples

Two numerical examples are taken from the literature to illustrate the algorithm of the proposed method. The reduction procedure is described for getting second order reduced model in the both examples. An integral square error [1] in between the transient parts of the original and the 2nd order reduced model is calculated using Matlab to measure the goodness of the reduced order model i.e. lower the ISE, closer the to $R_2(S)\ G_n(S)$, which is given by

$$\text{ISE} = \int_0^{\infty} [y(t) - y_2(t)]^2 dt \qquad (8)$$

Where, $Y(t)$ and $Y_2(t)$ are the unit step responses of original and 2nd order reduced system respectively.

Example.1 Consider a fourth order system [7] described by the transfer function

$$G_4(s) = \frac{N(s)}{D(s)} = \frac{24 + 24s + 7s^2 + s^3}{24 + 50s + 35s^2 + 10s^3 + s^4}$$

Let, a second order model is required then the following steps are as under:

Using the Step-1, the reciprocal of is taken as

$$\dot{D}(s) = s^4 D\left(\frac{1}{s}\right) = 1 + 10s + 35s^2 + 50s^3 + 24s^4$$

Now $\dot{D}(s)$ is differentiated twice and then resulting polynomial reciprocated back which gives

$$D_2(s) = 4.1143 + 4.2857s + s^2$$

Using the Step-2 of the proposed method, the second order model can be taken as

$$R_2(s) = \frac{N_2(s)}{D_2(s)} = \frac{c_0 + c_1 s}{(s + 1.4518)(s + 2.8339)}$$

In order to match steady-state values of 2nd order reduced model and the original system, the coefficient c_0 is taken equal to 4.1143. The coefficient c_1 is computed by minimizing the error (ISE) function 'J' using GA tool box of the Matlab 7.2 with the following typical parameters shown in the Table.1.

The GA with above parameters results. $c_1 = 0.014948$.

Therefore, finally second order model is given as $R_2(s) = \dfrac{4.1143 + 0.014948s}{4.1143 + 4.2857s + s^2}$ with

ISE : 4.266×10^{-3}.

Table1: Typical Parameter Used By GA

Name of Parameter	Type/Value
Population type	Double vector
Population Size	20
Selection	Stochastic function
Reproduction	Elite Count=2;C.F=0.8
Mutation	Gaussian; Shrink=0.292
Crossover function	Arithmetic
Generation	200

The step responses of 2nd reduced order model with the original system are plotted and shown in the Fig.2.

The comparison of the proposed method with the other well known order reduction methods is done by calculating integral square error (ISE) between the original and reduced system and tabulated in the Table.2.

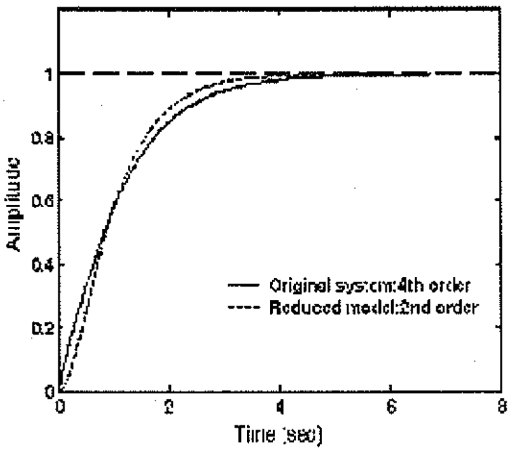


Figure 2 : Comparison Of The Step Responses

Example.2 Consider an eight-order SISO system [7]:

$$G_8(s) = \frac{N(s)}{D(s)} \quad \text{where}$$

$$N(s) = 40320 + 185760s + 222088s^2 + 122664s^3 + 36380s^4 + 5982s^5 + 514s^6 + 18s^7$$

and

$$D(s) = 40320 + 109584s + 118124s^2 + 67284s^3 + 22449s^4 + 4536s^5 + 546s^6 + 36s^7 + s^8$$

The reduction procedure for getting the second order reduced model is described, which is as under:

Using the Step-1, the denominator polynomial is obtained as

Method of reduction	Reduced model	ISE
Proposed method	$\frac{4.1143 + 0.014948s}{4.1143 + 4.2857s + s^2}$	4.266×10^{-3}
G. Parmar et. al [13]	$\frac{0.6991576 + 0.7442575s}{0.6997 + 1.45771s + s^2}$	1.744×10^{-3}
Pal [19]	$\frac{34.2465 + s}{34.2465 + 239.8082s + s^2}$	1.534272
Parthasarathy and J. [20]	$\frac{0.6997 + s}{0.6997 + 1.45771s + s^2}$	34.014×10^{-3}
Davison [21]	$\frac{2 - s^2}{2 + 3s + s^2}$	220.238×10^{-3}
Prasad and Pal [22]	$\frac{34.2465 + s}{34.2465 + 239.8082s + s^2}$	1.534272
Krishnamurthy and Seshasdri [23]	$\frac{24 + 20.5714s}{24 + 42s + 30s^2}$	9.5891×10^{-3}

Table 2 : Comparison Of Reduced Order Models (Example.1)

and using the Step.2 of the proposed method, the second order model is determined as $R_2(s) = \frac{9.55741 + 18.39954s}{9.55741 + 6.49392s + s^2}$

The convergence of the objective function 'J' is shown in the Fig.3 and the comparison of the time responses of the original and reduced system are shown in the Fig.4.

The comparison of the proposed method with the other well known order reduction methods is done by comparing integral square error (ISE) between the original system and the reduced 2nd order model and is shown in the Table.2.

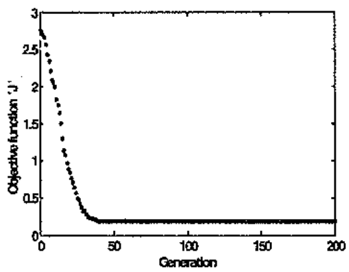


Figure 3 : Convergence Of The Objective Function (Example.2)

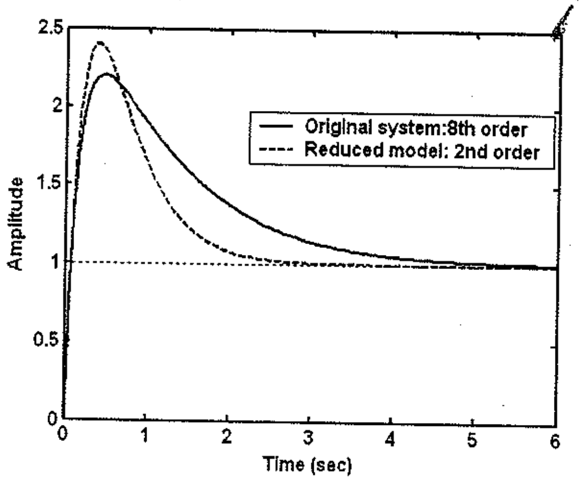


Figure 4: Comparison Of The Step Responses (Example.2)

Table 2 : Comparison Of The Reduction Methods (Example.2)

Reduction method	Reduced model	ISE
Proposed method	$\frac{9.55741+18.39954s}{9.55741+6.49392s+s^2}$	0.1948
Mittal et. al [24]	$\frac{1.9906+7.0908s}{2+3s+s^2}$	0.2689
Pal [25]	$\frac{40320+151776.576s}{40320+75600s+65520s^2}$	1.6509
Chen et al. [26]	$\frac{0.36669+0.72046s}{0.36669+0.02768s+s^2}$	7.2101
Prasad and Pal [8]	$\frac{500+17.98561s}{500+13.24571s+s^2}$	1.4584

4. Conclusions

A method for reducing the order of the large-scale SISO systems by using the polynomial derivative and Genetic Algorithm has been presented. In this method, the denominator polynomial is determined by polynomial derivative while the numerator coefficients are computed by minimizing the integral square error using Genetic Algorithm. This method is conceptually simple and computer oriented. The method has been applied on two problems to get second order reduced models. This method has been compared with the several order reduction techniques available in the literature and shown in the Table.1 and Table.2, from which it is clear that the proposed method is comparable in quality. This method also guarantees stability of the reduced order models if the original high-order system is stable and matches the steady-state value with the original high-order system. The proposed method is equally applicable to linear multivariable systems and is reported elsewhere.

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Author's Biography

C.B. Vishwakarma was born in Kanpur (U.P), India, in 1975. He received B.E. degree in Electrical Engineering from University of Roorkee, India, in 1998 and M.Tech in System Engineering and Operation Research from the Department of Electrical Engineering at Indian Institute of Technology Roorkee, India, in 2002. Since then, he is working as a lecturer in Faculty of Engineering and Technology, Gurukul Kangri University, Haridwar, India. Presently he is also Research scholar (part-time) in the Department of Electrical Engineering at Indian Institute of Technology Roorkee (India).

Dr. Rajendra Prasad was born in Hangawali (Saharanpur), India, in 1953. He received B.Sc.(Hons.) degree from Meerut University, India, in 1973. He received B.E., M.E. and Ph.D degrees in Electrical Engineering from University of Roorkee, India, in 1977, 1979, and 1990 respectively. He served as Assistant Engineer in M.P.E.B. from 1979-1983. From 1983 to1996,he was a lecturer in the Electrical Engineering Department of University of Roorkee, Roorkee (India) and from 1996-2001, he was Assistant Professor. Presently, he is an Associate Professor in the Department of Electrical Engineering at Indian Institute of Technology Roorkee (India). His research interests include Control, Optimization, System Engineering and Model order reduction of large scale systems.