

# THE EFFECTS OF INCLINED MAGNETIC FIELD AND INCLINED PLATE ON MHD CASSON FLUID

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## Abstract

The current note comprises the impact of inclined attractive field and inclined plate on MHD stream of a Casson liquid over a vertical permeable plate. The mathematical analysis is carried out analytically by using the series solution method. The physical highlights of different parameters on the flow have been examined and delineated by the graphs.

**Keywords:** MHD casson fluidmagnetic field, velocity, temperature.

## I INTRODUCTION

During the last few decades, Casson fluid has interesting applications in biological, chemical, medical and engineering applications. It exhibits yield stress. Tomato sauce, honey, soup, jelly, and human blood, etc. are some examples of Casson fluid. Several researchers have investigated in this direction. Some of them are Raju et.al [1], Jasmine et.al [2], Reddy [3], Rashidi [4], Ajayi [5], Ramana Reddy [6], Eswara Rao [7], Shalini Jain and Amit Parmar [8], Gireesha et.al [9], Kataria and Patel [10], Khalid [11], Animasaun [12], Eegunjobi [13], Biswas [14], Abro et.al [15], Arifuzzaman et.al [16], Umamaheswar [17], Balaji Prakash et.al [18], Manjula et.al [19], Prasad et.al [20].

Spurred by the above writing, we endeavour to examine the consequences of attractive slanted field and slanted permeable plate on MHD Casson fluid over a vertical permeable plate diagnostically and introduce the impacts of fluctuating speed parameters temperature fields with the

guide of graphs.

## II MATHEMATICAL ANALYSIS

We study an in-compressible Casson fluid's two-dimensional free convection and mass trade stream past an immense vertical porous plate. The attractions speed typically to the plate is steady and can be composed as,

$$V^I = -U_0$$

A course of action of rectangular co-ordinate  $O(x^I, y^I, z^I)$  is taken, with the ultimate objective that  $y^I = 0$  on the plate and  $z^I$  rotate is along its driving edges. All the fluid properties are considered predictable. Besides, the effect of the thickness assortment with temperature is thought of. The effect of the thickness assortment in various terms of the power and essential condition and the augmentation coefficient with temperature is seen as irrelevant. This is the prominent Boussinesq estimate.

The Governing equations are

Continuity equation :

$$\frac{\partial V^I}{\partial y^I} = 0 \quad (1)$$

Momentum equation:

$$\frac{\partial u^I}{\partial t^I} + V^I \frac{\partial u^I}{\partial y^I} = g\beta(T^I - T_\infty^I) \cos \alpha + \nu(1 + \frac{1}{\beta}) \frac{\partial^2 u^I}{\partial y^{I^2}} - \left[ \frac{\sigma B_0^2 \sin \beta}{\rho} + \frac{\nu}{K^I} \right] u^I \quad (2)$$

Energy equation:

$$\frac{\partial T^I}{\partial t^I} + V^I \frac{\partial T^I}{\partial y^I} = \alpha \left( \frac{\partial^2 T^I}{\partial y^{I^2}} \right) + Q_0(T^I - T_\infty^I) \quad (3)$$

Where  $u^I, V^I, T^I, C^I, \alpha$  and  $Q_0$  represents velocity, temperature, concentration thermal conductivity and dimensional heat absorption coefficient.

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The associated conditions are:

$$\begin{aligned} u^I &= 0, T^I = T_w^I, C^I = C_w^I \text{ at } y^I = 0 \\ u^I &= 0, T^I = T_\infty, C^I = C_\infty \text{ at } y^I \rightarrow \infty \end{aligned} \quad (4)$$

Let us introduce the non-dimensional variables.

$$\begin{aligned} u &= \frac{u^I}{U_0}, t = \frac{t^I U_0^2}{\nu}, y = \frac{y^I U_0}{\nu}, T = \frac{T^I - T_\infty^I}{T_w^I - T_\infty^I}, \\ K &= \frac{K^I U_0^2}{\nu^2}, P_r = \frac{\nu}{\alpha}, M = \frac{\sigma B_0^2 \sin \beta \nu}{\rho U_0^2}, B = 1 + \frac{1}{\beta}, \\ Q &= \frac{Q_0 \nu}{\rho c_p U_0^2}, G_r = \frac{\nu g \beta (T_w^I - T_\infty^I)}{U_0^3} \end{aligned} \quad (5)$$

Where  $P_r$  - Prandtl number,  $G_r$  - Grashof number,  $N$  - buoyancy ratio,  $B$  - Casson parameter,  $M$  - inclined magnetic parameter,  $K$  - permeability parameter,  $\beta$  - thermal expansion coefficients,  $\beta^I$  - concentration expansion coefficient and  $Q_0$  - dimensional heat absorption coefficient.

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = G_r T \cos \alpha + B \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right) \quad (6)$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} + (Q + R)T \quad (7)$$

and

$$\begin{aligned} u &= 0, T = 1 \text{ at } y = 0 \\ u &= 0, T = 0 \text{ at } y \rightarrow \infty \end{aligned} \quad (8)$$

### III METHOD OF SOLUTION

Assume that  $u$  and  $T$  as

$$\begin{aligned} u(y, t) &= u_0(y) e^{-nt}, \\ T(y, t) &= T_0(y) e^{-nt} \end{aligned} \quad (9)$$

Using equation (9) in equation (6) and (7) we get

$$B u_0'' + u_0' - \left[M + \frac{1}{K} - n\right] u_0 = -G_r T_0 \cos \alpha \quad (10)$$

$$T_0'' + P_r T_0' + P_r (n + Q) T_0 = 0 \quad (11)$$

$$u_0 = 0, T_0 = 1 \text{ at } y = 0$$

$$u_0 = 0, T_0 = 0 \text{ at } y \rightarrow \infty \quad (12)$$

Solving the equations (10) and (11), using boundary conditions (12), we get the following equations.

$$T_0 = e^{-m_1 y} \quad (13)$$

$$u_0 = (A_1) e^{-m_2 y} - A_1 e^{-m_1 y} \quad (14)$$

where,

$$m_1 = \frac{P_r + \sqrt{P_r^2 - 4P_r(n + Q)}}{2}$$

$$m_2 = \frac{1 + \sqrt{1 + 4\left(M + \frac{1}{K} - n\right)}}{2}$$

$$A_1 = \frac{G_r}{B m_1^2 + m_1 - \left(M + \frac{1}{K} - n\right)}$$

Hence the Solutions of  $u$  and  $T$  will be as follows

$$u = [(A_1) e^{-m_2 y} - A_1 e^{-m_1 y}] e^{-nt} \quad (15)$$

$$T = e^{-m_1 y} e^{-nt} \quad (16)$$

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = [-m_2(A_1) + m_1 A_1] e^{-nt} \quad (17)$$

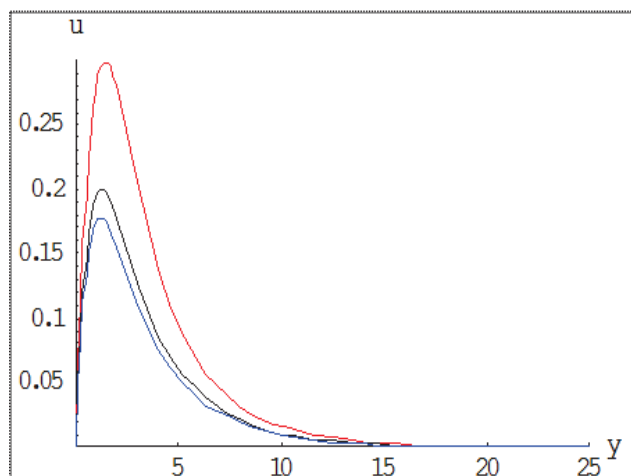
Where  $\tau$  is skin friction

### IV RESULTS AND DISCUSSION

This paper has contemplated the inclined plate and magnetic field on MHD Casson fluid with heat absorption/generation. The graphs have concentrated the impact of the parameters  $\beta$ ,  $G_r$ ,  $K$ ,  $P_r$ ,  $B$ ,  $\alpha$  and  $Q$  on stream attributes.

To have physical relationships, we pick reasonable estimations of flow parameters. The graphs of speed, temperature and mass focus are taken regarding  $y$ .

**Velocity profiles:** At  $n = 0.1$  and  $t = 0.1$ , the Velocity profiles are depicted in Figures 1 – 7.

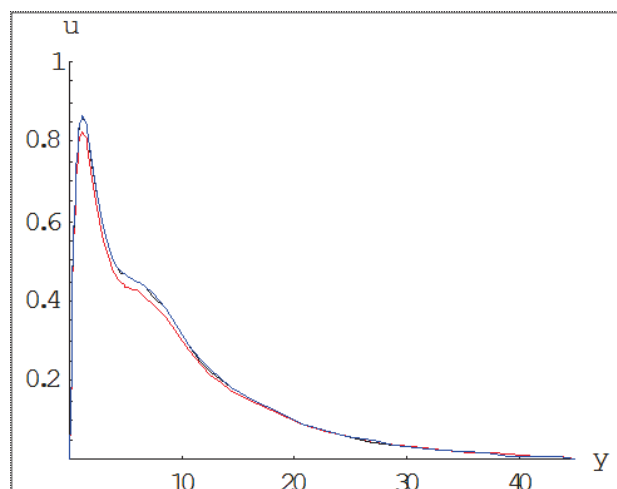


**Figure 1**

Effects of inclined magnetic field on u profiles

When  $Pr = 0.71$ ,  $Gr = 1$ ,  $Q = 1$ ,  $B=1$  and  $K = 10$

Figure - 1 shows the impact of the inclined Magnetic field on the Velocity profiles. speed diminishes with the expansion in the inclined attractive parameter ( $\beta$ ).

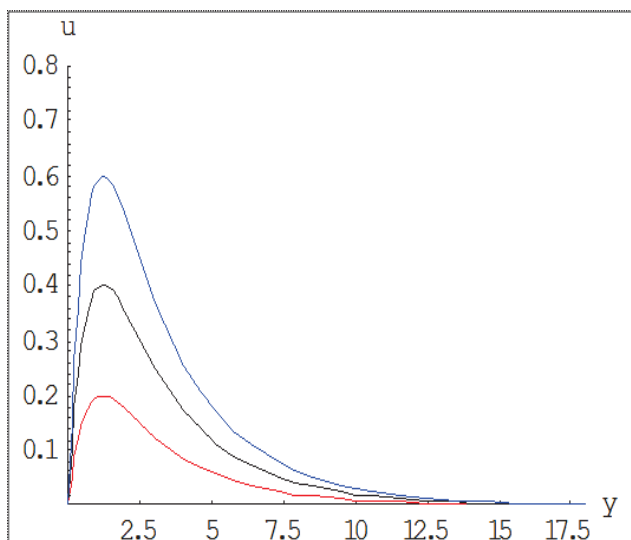


**Figure 3**

Effects of k on velocity profiles

When  $Pr = 0.71$ ,  $Gr = 1$ ,  $Q = 1$ ,  $B=1$  and  $M = 2$

Figure - 3 shows Changes of Permeability parameter K on the Velocity profiles. Here speed increments with porousness parameter (K).

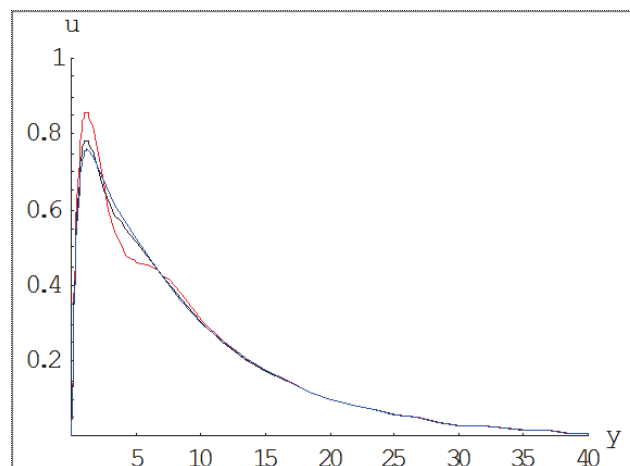


**Figure 2**

Effects of Gr on velocity profiles

When  $Pr = 0.71$ ,  $M = 1$ ,  $Q = 1$ ,  $B=1$  and  $K = 10$

Figure - 2 shows varieties of Grashoff number Gr on the Velocity profiles. It delineates that speed increments with raise in Grashoff number (Gr).

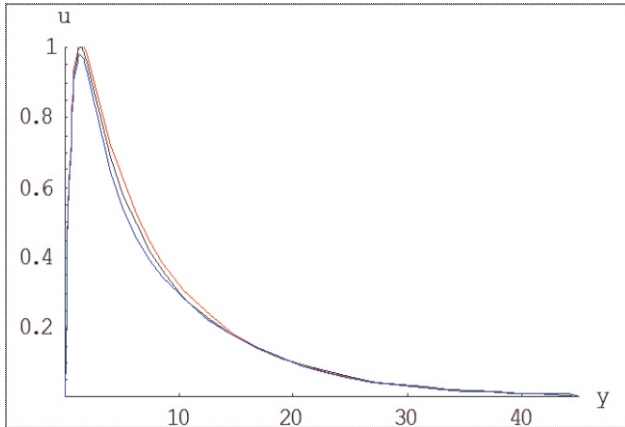


**Figure 4**

EEffects of Pr on velocity profiles

When  $Gr = 1$ ,  $Q = 1$ ,  $M = 2$ ,  $B=1$  and  $K = 100$

Figure – 4 shows the impact of the Prandtl number Pr on the Velocity profiles. Here, the Prandtl number (Pr) adds to the speed increment close to the beginning and lessening endlessly from the root.

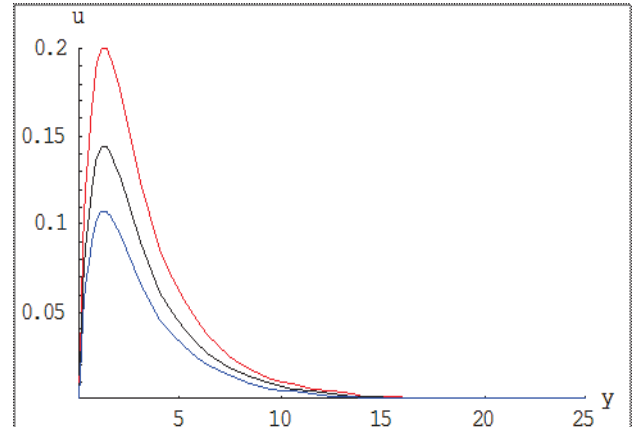


**Figure 5**

Effects of  $Q$  on velocity profiles

When  $Pr = 0.71$ ,  $Gr = 1$ ,  $M = 2$ ,  $B = 1$  and  $K = 10$

Figure - 5 shows the Heat assimilation coefficient  $Q$  impact on the Velocity profiles. It is seen that an ascent in the warmth ingestion coefficient ( $Q$ ) adds to decreasing in the speed.

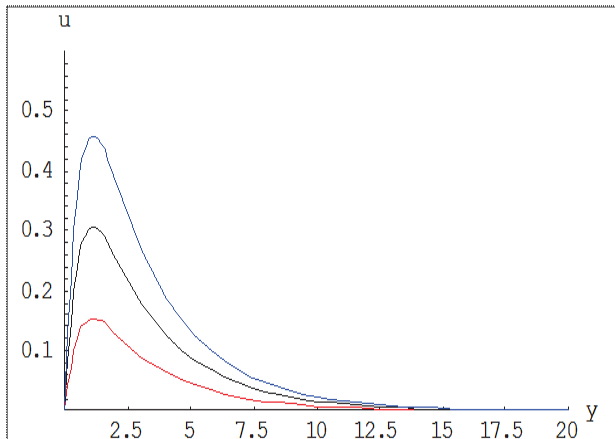


**Figure 7**

Effects of  $B$  on velocity profiles

When  $Pr = 0.71$ ,  $Gr = 1$ ,  $M = 2$ ,  $B = 1$  and  $K = 10$

Figure - 7 shows the impact of the Casson parameter  $B$  on the Velocity profiles. It is seen that expansion in the Casson parameter  $B$  adds to diminish in the speed.

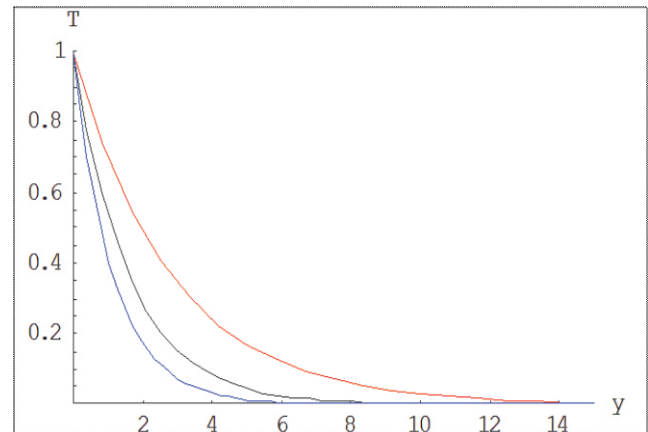


**Figure 6**

Effects of inclined porous plate on  $u$  profiles

When  $Pr = 0.71$ ,  $Gr = 1$ ,  $M = 2$ ,  $B = 1$  and  $K = 10$

Figure - 6 shows the slanted permeable plate impact  $\alpha$  on the Velocity profiles. It is seen that an ascent in  $\alpha$  adds to falls in the speed.

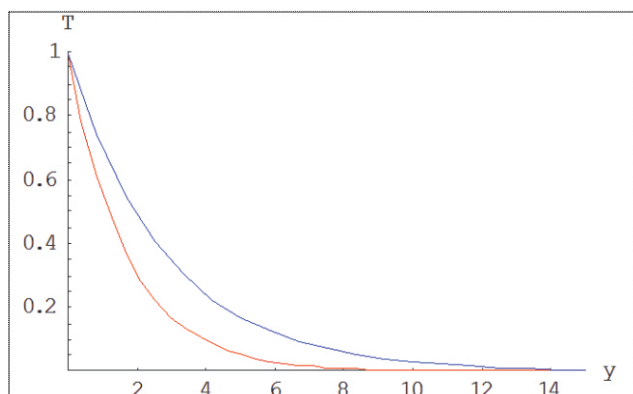


**Figure 8**

Effects of  $Pr$  on Temperature profiles

When  $Gr = 1$ ,  $Q = 1$ ,  $M = 2$  and  $K = 10$

Figure - 8 shows the impact of the Prandtl number  $Pr$  on the Temperature profiles. It is seen that expansion in the Prandtl number ( $Pr$ ) adds to diminish the temperature in the liquid medium.



**Figure 9**

Effects of  $Q$  on Temperature profiles

When  $Pr = 0.71$ ,  $Gr = 1$ ,  $M = 2$  and  $K = 100$

Figure - 9 shows the impact of the Heat retention parameter  $Q$  on the Temperature profiles. It is seen that expansion in the Heat ingestion parameter ( $Q$ ) increases the temperature in the liquid medium.

## V CONCLUSION

This paper breaks down the attractive slanted field and slanted permeable plate impacts on MHD Casson liquid stream past a vertical permeable plate with heat assimilation or age. The non-dimensional administering conditions were fathomed by irritation strategy. Finishes of the investigation can be given as follows:

1. Velocity diminished with expanded estimations of slanted attractive parameter  $M$ , Heat assimilation coefficient  $Q$ , Casson parameter  $B$  and Prandtl number  $Pr$ .
2. Velocity increased as the parameter of Grashof number  $Gr$ , permeability parameter  $K$ , inclined porous plate  $\alpha$ , and Prandtl number  $Pr$  increased.
3. Temperature diminished as Prandtl number  $Pr$  expanded and expanded with heat retention coefficient  $Q$  expansion.

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