COMPARISON OF HYPERGEOMETRIC DISTRIBUTION WITH POISSON DISTRIBUTION IN SUPPLY CHAIN - A SIMULATION STUDY

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Abstract

In the acceptance sampling, several published research papers dealt with the performance of various sampling plans based on operating characteristic function and other related measures only. In this paper, an attempt is made to compare the performance of Hyper-geometric distribution and Poisson distribution under 100% Screening Policy in the SCM in terms of the cost-effectiveness in the supply chain using the simulation software Goldsim 12.0. The study found a little difference only between the distributions and sampling plans in the overall supply chain system.

Keywords: Acceptance Sampling, Economic Order Quantity, Inspection, Quality, Simulation, Supply Chain Management.

1 INTRODUCTION

ANSI/ASQC standard A2 (1987) [1]defines acceptance sampling as the methodology which deals with procedures from that decision can be made whether to accept or reject a lot based on the results of the sample inspection. In acceptance sampling, an inspection of items can vary from 100% of the delivery to a relatively few items from which the receiving firm draws inferences about the whole shipment.

To separate unwanted lots from good ones, it is to give attention to the following aspects i) simple administrative procedures, ii) economy of observations, iii) increase the lot size to reduce the risk, iv) from a valuable source of information to use accumulated sample data, v) more importantly the sample size should be reduced (Hamaker, 1960) [2] when the quality is reliable and maintained at a satisfactory level.

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In this paper, the non-Bayesian approach is applied for a lot-by-lot sampling plan in supply chain systems.

II REVIEW OF LITERATURE

This study discusses the acceptance sampling method through Single Sampling and Double Sampling plans. An attempt is made to integrate inventory management in supply chain management with acceptance sampling.

In any inventory management system, the Economic Order Quantity (EOQ) formula is probably one of the most famous formulae in the industrial management field; the composition and estimation of the cost parameters have always been vague and imprecise at best. Because of the inherently, the fuzzy aspect of the cost determination and the EOQ formula are re-examined in a fuzzy-set-theoretic perspective (Park, 1987)[3].

In this study, a comparative analysis is made to investigate the effects of five important parameters (inspection rate, the demand, the defective rate, the holding cost, and the ordering cost[4] (as given in Lin, 2013)). Shanmugam (1985) introduced[5] Intervened Poisson Distribution (IPD) derived from Zero Truncated Poisson Distribution (ZTPD) and used by Radhakrishnan and Sekkizhar (2007)[6] and Sekkizhar (2008)[7] in designing various sampling plans.

Hyper-geometric Distribution

Samohyl (2018) [8] has shown that sampling plans should be valid to the data and the situation they represent. Statistical approximations may lead to serious estimation errors when lot sizes are finite. Including lot size, the sampling process should employ only the most accurate data available. The priority for researchers should be the utilization of the most appropriate formulation. The part of the Hyper-geometric distribution is given by

$$p(x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}} \quad \ldots \quad (1)$$

where,

N - lot size (or Economic Order Quantity Q)

x - no. of defectives observed during the sample inspection

n - Sample size

m - No. of defectives in the lot (if proportion defective (p) is given, thenm=Np).

Poisson Distribution

For modelling any rare events, an ancient probability distribution - Poisson Distribution [9,10] is used by several authors to construct various sampling plans. The probability mass function of the Poisson random variable Z is given by

$$P(Z = z) = \frac{e^{-\theta}\theta^{z}}{Z!}, \quad z = 0, 1, 2, \ldots. \quad \ldots.. \quad (2)$$

with mean = variance = θ.

2.1 The objective of the study

The main objective of this study is to verify whether the Hyper-geometric distribution is cost-effective than Poisson distribution in the overall supply chain process.

III RESEARCH METHODOLOGY

In this study, the simulation technique is used to design the models for single and double sampling plans in supply chain management to determine the Economic Order Quantity. Various costs are taken into account to compare the two plans to determine the cost-effectiveness using simulation models.

Gold Sim (12.0) is specifically designed simulation-based software to address the inherent uncertainty in real-world systems quantitatively. It is user-friendly and well supported for graphical presentation. In addition, it provides powerful capabilities to cover up the happening of discrete events into continuously varying systems. In this study, GoldSim simulates discrete events such as financial transactions and the number of resources utilized.

3.1 Supply Chain Model

In this paper, the re-ordering system assumes the items submitted for production process or purchase by the people. However, practically there are possibilities of getting defective items i.e., defective or it might not meet the required production process standards. Therefore, there is a need for rectification of the defective/imperfect items. Hence, the researcher applies acceptance sampling in-between (during the production process) the purchase and production process stage. Generally after purchase, the entire lot has been sent for the production process. But in this proposed model, the purchased lot is being sent for sampling inspection. At this stage, if the acceptance sampling results are positive and if the lot is accepted, then the lot is being sent to the production process. If the lot is rejected, then it will be sent to 100% screening. Moreover, after that, the lot will be again sent to the previous stage. In some situations, the lot will be sent back to the supplier itself for replacement, but this situation is not considered in this study.

In the proposed model, if there is any defective found, say for example 1%, that particular defective lot is being sent for sampling inspection. Sampling inspection will decide whether the lot has to be sent for direct production or undergo screening. If the lot is being sent for production directly, the finished product will also reflect the same percentage of defective, which was detected in the raw material lot. This means the production level has to be increased, i.e. instead of producing 100 items, 101 items have to be produced.

They are considering another situation where the defective item is being sent for screening. The screening is good because all the defective items will be replaced. Therefore, after the production process, the finished goods will be free from defectiveness. However, 100% screening policy is a bit costlier when compared to sampling inspection. So, the decision will be taken by considering whether the lot has to be sent for inspection or not. If the lot is sent for inspection, it has to undergo a rectification process which involves rectification cost. If the lot is sent for inspection, it incurs just the inspection cost. If the customer is receiving a defective lot, it incurs warrant cost, and producing extra items again incurs manufacturing costs. All these aspects need to be considered for having a good production process. However, if analytical models are considered for deriving at some equations, it becomes more complicated since we need to consider more parameters. Hence, the researcher is trying to solve this with the simulation model. In the simulation model, all the parameters are being fixed with numerical values to determine the total cost of implementation and the optimal model is being determined.

Gold Sim software is being used because this model involves inventory cost, storage cost (if the produced lot has to be kept inside the go down, it involves cost for storage), backorder cost (if there is a shortage of goods or if there is any delay in delivery of the lot) and ordering cost. The GoldSim can do the computations of time durations, cost and inventory levels, etc.

3.2 100% Screen Policy Model

A supply chain process is given in the Figure-1, which shows the supply chain process i.e. from purchase, the lot is being sent to sampling inspection, after which the lot is being sent to the production process (the arrows are used just for referencing and not to show the flow of the process).

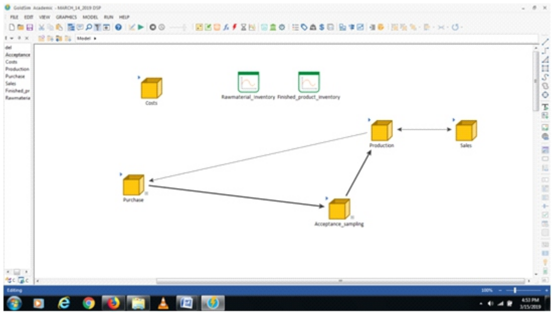

Figure-1: Operating Procedure for the supply chain system under study

Finally, the lot will be sent to sales after production process. Purchase decisions are taken based on the production unit. The level of production will be determined based on the availability of the raw material. If there is any change, it is called the re-order system, i.e. if stock level is less than the expected level, there will be a need for a new purchase order.

IV COST INPUTS

For this simulation process, we must input specific values for the various costs involved in the supply chain. Therefore from various research papers and few qualitative interviews with experts, the researchers were able to find out the cost patterns. This pattern[11] is taken from Hlioui et. al (2015) and is presented in the Table-1.

*Table-1: Inputs for Various Costs**

Type	Item of cost	Cost (Rs)
Raw Material	Material cost	0.5
Finished product	Manufacturing cost	0.5
Raw Material	Storage cost	1
Finished product	Storage cost	1
Raw Material	Inspection Cost	12
Finished product	Backorder(Shortage) cost	40
Raw Material	Rectification cost	65
Finished product	Replacement (warranty) cost	90
Fixed	Ordering Cost	300

Source: Hliouiet. al (2015)

The material cost of raw material is 50%, the manufacturing cost of the finished product is 50%, which is like 50 paise, 50 paise (0.5, 0.5). The storage cost of raw material and the finished product is Re.1 each, per unit per day. The inspection cost of raw materials is Rs. 12, the backorder cost is Rs. 40, rectification cost for raw material is Rs. 65, which involves inspection, changing cost, etc. The replacement cost for the finished product is Rs. 90, which involves warranty, delivery charges, etc. The cost of the product is Re. 1, but when it reaches the customer, they have to pay Rs. 90, which involves all the related costs. When any optimization is being done in the future, automatically, the number of defective is expected to be low. There might be some stress to reduce the number of defectives. Hence, this is the finished cost for the product.

V COMPARISON BETWEEN SAMPLING PLANS CONSTRUCTED USING HYPER-GEOMETRIC DISTRIBUTION AND POISSON DISTRIBUTION UNDER 100% SCREENING POLICY FOR REJECTED LOT

This paper attempts to compare Hypergeometric distribution with the Poisson distribution under 100% Screening Policy to identify how the Hyper-geometric distribution works in supply chain management. For this purpose, a mock script has been created using the GoldSim software. Samohyl (2018) found that acceptance sampling plans for attribute used Hyper-geometric distribution and emphasized that it is more critical than Binomial and Poisson distributions. The Hyper-geometric distribution is used in small sample cases. Further, if the proportion defective (p) is closer to 0.05 or even 0.01, 0.02 it might work effectively.

Algorithm for Hyper-geometric Distribution

In this section, the input parameters for the simulation model, namely, the demand (D), re-order point (Q), sample size (n) and acceptance number (c) while estimating costs using Hyper-geometric and Poisson distributions, have been taken as 20, 100, 50 and 3 as inputs for. Furthermore, the costs for various proportions of defective (p) are simulated and presented in Table 2.

p - value ⟶	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Total Cost for Hyper-geometric	161117	160970	163649	163624	168222	168363	168363
Total Cost for Poisson	186233	195205	207805	222777	237727	252677	267627

Table 2:Total Costs simulated using Hyper-geometric and Poisson distributions

From Figure 2, it is found that the cost due to the Poisson distribution is higher than that of Hyper-geometric distribution, which is similar to the study conducted by Samohyl (2018). As the total cost due to Hyper-geometric distribution is lesser than the Poisson distribution[12], the Hyper-geometric distribution is more appropriate than the Poisson distribution. It is applicable only for the cases where the values are 0.2, 0.4, 0.6, 0.8. It is meaningless in the present-day industry because defectives of 20%, 30%, or 40% are not acceptable. At present, the industries are working towards zero defective or meagre percentage of defectives. Hence, both Poisson and Hyper-geometric distributions may give more or less the exact cost.

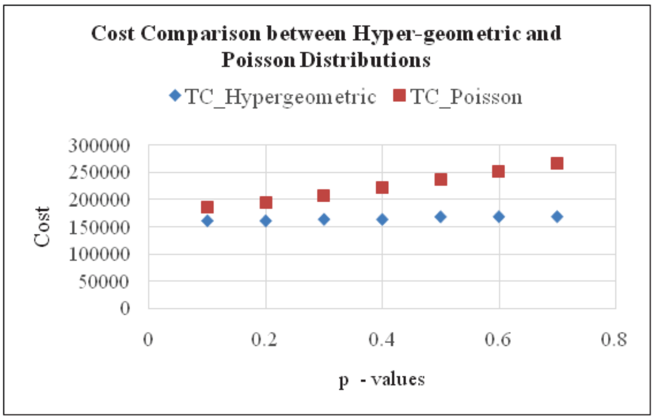

Figure 2: Cost Comparison between Hyper-geometric and Poisson Distributions

Therefore, in order to find out the significance[13] of the difference between the total costs due to Poisson distribution and Hyper-geometric distribution-oriented supply chain, a paired t-test is used to test the null hypothesis that there is no significant difference between the total costs due to the Hyper-geometric and the Poisson distributions[14] and the test results are presented in Table 3. It is found that there is a significant difference between the total costs due to Hyper-geometric and the Poisson distributions.

Table 3: Paired t-Test for the Mean costs of Hyper-geometric and Poisson Distribution

Measures	Total Costs	
	Using Hyper-geometric	Using Poisson
Mean	164901.1	224293
Standard Deviation	3365.75	30083.23
Number of Observations	7	7
Karl Pearson Correlation Coefficient	0.9443	
Hypothesized Mean Difference	0	
degrees of freedom (d.f)	6	
t - statistic value	-5.83383	
Probability ($T \leq t$) for one -tail test	0.000559*	

** Significant at 1%*

Further more, it is concluded by considering Proposition (P1) that the Hyper-geometric distribution is more effective than the Poisson distribution for Lot Size Determination[15]. There is a considerable cost reduction due to Hyper-geometric distribution for greater the proportion defective (p) values.

VI CONCLUSION

Comparison is made to find out the efficient probability distribution in terms of cost-effectiveness in Supply Chain Management. In the acceptance sampling integrated with the supply chain, Hyper-geometric distribution is more effective than the Poisson distribution for Lot Size Determination in case of smaller proportion defective values. However, for larger lots, hyper-geometric distribution has its computational difficulties.

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