# Wavelet Packet Image Compression using Log Energy Entropy Cost Function

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## Abstract

This paper demonstrates compression of an image using wavelet packet decomposition. One reasonable criterion for selecting decomposition for the compression of image is the additive cost function. The entropy based cost function used here to select the best basis is the log-energy entropy metric. The algorithm checks the entropy of decomposed nodes (child nodes) with entropy of node, which has been decomposed (parent node) and takes decision of decomposition of a node. An adaptive thresholding is used for quantization. The wavelet packet coefficients are classified as important or unimportant depending on whether its value lies above or below the threshold value. After the data has been quantized into a finite set of values, it can be encoded using an entropy coder to give additional compression. Results are compared in terms of encoding time, decoding time, signal to noise ratio, root mean square error value and compression ratio.


**Keywords:** Image Compression, Wavelet Packet Tree, Best Basis, Log Energy Entropy

## 1. Introduction

Image compression is very important in many applications, especially for progressive transmission, image browsing and multimedia applications. No single image compression algorithm can be expected to work well for all classes of digital images. The sampling rates, frequency content, and pixel quantization all influence the compressibility of the original data. Subsequent machine or human analyses of the compressed data, or its presentation at various magnifications, all influence the nature and visibility of distortion and artifacts. Algorithms based on wavelets have shown to work well in image compression. Advances in wavelet transform and quantization methods have produced the algorithm capable of surpassing the existing image compression standards like the Joint Photographic Experts Group (JPEG Algorithm).

Wavelet transform is modeling the signals by combining translations and dilations of a simple oscillatory function of finite duration called a wavelet. Wavelet transform is used for analysis of the image at different decomposition levels. These decomposition levels contain a number of subbands, which consists of coefficients that describe the horizontal and vertical spatial frequency characteristics of the input image.

Despite of general success, the wavelet transform often fails to accurately capture high frequency information, especially at lower bit rates where such information is lost in quantization noise. A technique has been developed called wavelet packets that are better able to represent high frequency information. Wavelet packets are the conventional wavelet transforms in which the details are iteratively filtered, i.e. the signal is passed through more filters than DWT.

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This paper is organized as follows: In Section 2 wavelet packets are described. In Section 3 selection of best wavelet packet basis is explained, quantizing the wavelet packet coefficients in section 4. Coding results are presented in Section 5, followed by conclusion in Section 6.

## 2. Wavelet Packets

In the Discrete Wavelet Transform (DWT), a signal may be represented by its approximations and details. The approximation is the low frequency component of the signal. The detail is the high frequency component. Each level is calculated by passing the previous approximation coefficients through a high and low pas filters. In Wavelet Packet Decomposition (WPD), both the detail and approximation coefficients are decomposed. In Figure 1, 'S' represents the raw signal, 'A' represents the approximations and 'D' represents the details. A signal can be broken into lower-resolution components by using different scales. This is the wavelet decomposition tree. The wavelet packet decomposition tree is a part of the complete binary tree.

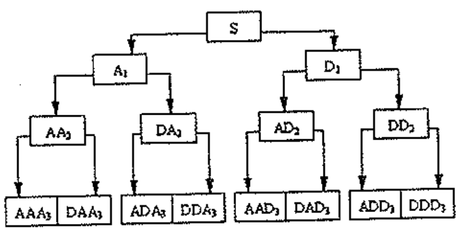



**Figure 1-Wavelet packet decomposition tree**

Decomposing into a sequence of wavelet packet coefficients is done using wavelet packet transform. The wavelet packet transform [1] can be viewed as a tree. The root of the tree is the original data set. The next level of the tree is the result of one step of the wavelet transform. Subsequent levels in the tree are constructed by recursively applying the wavelet transform step to the low and high pass filter results of the previous wavelet transform step. The wavelet function used here to construct the wavelet packet tree is a version of the Haar wavelet.

## 3. Best Wavelet Packet Basis

The wavelet packet basis is constructed adaptively based on some cost function and choice of decomposition filter applied to an image. The notion of the *best* wavelet packet basis is limited to the cost function based on which the full wavelet packet tree is pruned to obtain an optimal tree. The decomposition of a signal can be viewed as a tree, where the left branch represents the low-pass horizontal/low-pass vertical filtering, the right branch represents the high-pass horizontal/high-pass vertical filtering and the middle branches represents the low-pass horizontal/high-pass vertical filtering and the high-pass horizontal/low-pass vertical filtering, respectively.

In order to achieve compression gain while keeping the computational load reasonably low, two entities are needed: a criterion (cost function) for basis comparison and a fast search algorithm, which finds the best basis from the set of all possible bases. The best basis can be selected using either entropy-based cost, as proposed in [2], or by jointly estimating the rate-distortion function, as in [6]. These methods work by fully decomposing all subbands to a predefined maximum depth, thus forming a decomposition tree where each decomposed subband is represented in the tree by a parent node with four child nodes. Then the best basis is found by pruning this decomposition tree in a recursive bottom-up fashion. The entropy-based technique prunes the tree to minimize the overall estimated entropy of the wavelet packet structure. The rate-distortion method is given a particular target bit rate for the image and prunes the tree to minimize the distortion of the image.

The wavelet basis is the range in the vector over which the scaling and wavelet functions are non-zero. For the Haar wavelet the basis is an increasing power of two, relative to the original data set. The best basis algorithm finds a set of wavelet bases that provide the most desirable representation of the data relative to a particular cost function. A cost function may be chosen to fit a particular application. Selection of best basis for any particular image may be performed in number of ways. Here an entropy based additive cost function is used to select the best basis called the log-energy entropy metric

$$\cos t_{\log} = \sum_i \log(v_i^{\,2})$$

where $v_i$ represent the value of the coefficients of a subband.

The best basis selection algorithm decides a decomposition structure among the library of possible bases, by measuring a data dependent *cost function*. To calculate the best basis, the tree is traversed and each node is marked with its cost value (relative to a particular cost function). When the wavelet packet tree is constructed, all the leaves are marked with a flag. These "marks" are modified in calculating the best basis set. The best basis calculation is performed bottom up (that is, from the leaves of the tree toward the root):

- A leaf (a node at the bottom of the tree with no children) returns its cost value.
- As the calculation recurses up the tree toward the root, if there is a non-leaf node, **v1** is the cost value for that node. The value **v2** is the sum of the cost values of the children of the node.
  - If (**v1** <= **v2**) then mark the node as part of the best basis set and remove any marks in the nodes in the sub-tree of the current node.
  - If (**v1** > **v2**) then the cost value of the node is replaced with v2.

Once the best basis has been selected according to the cost function, the image is represented by a set of wavelet packet coefficients. The wavelet packet coefficients are then organized by increasing frequency.

## 4. Quantization

Quantization refers to the process of approximating the continuous set of values in the image data with a finite (preferably small) set of values. Once the wavelet packet coefficients are properly ordered, quantize the coefficients. These coefficients are quantized during a number of iterations of what is called the Quantization loop. This loop generates the quantization code. Whenever the stopping condition is met the loop is terminated and with it the whole process. In each iteration of the quantization loop the wavelet packet coefficients are classified into two classes:- those coefficients whose magnitude is larger than the threshold are important, - the others are unimportant with respect to the current threshold. The initial threshold $T_0$ is set to be half of the maximal magnitude of all wavelet packet coefficients. The threshold is divided by two at each iteration, and as a result the group of important coefficients widens from one phase to the next. The list of bits emitted by the quantizer are entropy coded using a zero run length coding technique.

## 5. Experimental Results

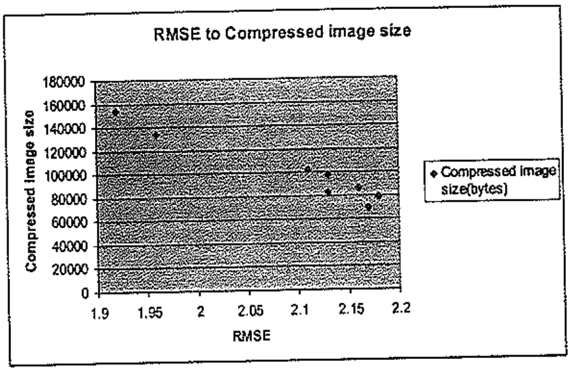
Data compression success is usually measured in the reconstructed image by the mean squared error (MSE), signal to noise ratio etc. although these global error measures do not always reflect subjective image quality. The coder has been applied to a variety of input images used by the image coding community. Testing has been performed on images of dimensions 512 x 512 and 256 x 256. Table 1 shows the PSNR values and compression ratios for various images. Table 2 gives the PSNR values for different bit rates for all the tested images.

**Table 1 –PSNR & CR for various images**

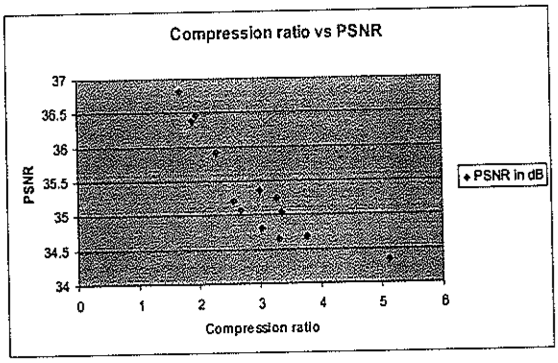
| Name of the images | PSNR in dB | Comp. Ratio |
|---|---|---|
| Barbara | 35.20 | 2.573 |
| Bird | 34.34 | 5.121 |
| Boat | 35.04 | 3.363 |
| Bridge | 36.38 | 1.900 |
| Cameraman | 35.23 | 3.277 |
| Frog | 36.46 | 1.958 |
| Goldhill | 35.06 | 2.680 |
| Lena | 35.36 | 3.005 |
| Mandril | 35.91 | 2.286 |
| Mountain | 36.82 | 1.698 |
| Peppers | 34.80 | 3.035 |
| Zelda | 34.68 | 3.774 |

**Table 2 – PSNR for different bit rates**

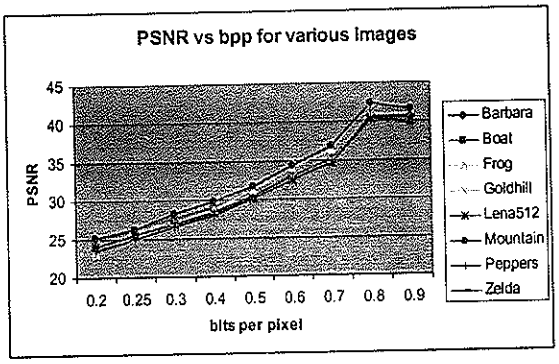
| Bit rate(b/p) | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 |
|---|---|---|---|---|---|
| Barbara | 24.00 | 27.04 | 28.48 | 30.50 | 32.98 |
| Bird | 26.39 | 28.44 | 29.58 | 30.79 | 32.47 |
| Boat | 24.79 | 27.68 | 29.08 | 30.86 | 33.08 |
| Bridge | 23.12 | 26.65 | 28.45 | 30.90 | 33.95 |
| Cameraman | 26.31 | 28.52 | 29.74 | 31.26 | 33.15 |
| Frog | 23.31 | 26.98 | 28.79 | 31.18 | 34.02 |
| Goldhill | 22.95 | 26.32 | 27.92 | 30.22 | 32.88 |
| Lena | 24.30 | 27.52 | 28.77 | 30.95 | 33.43 |
| Lena512 | 24.10 | 26.81 | 28.29 | 30.13 | 32.43 |
| Mandril | 22.85 | 26.56 | 28.23 | 30.77 | 33.42 |
| Mountain | 25.22 | 28.21 | 29.85 | 31.70 | 34.37 |
| Peppers | 23.56 | 26.58 | 28.14 | 30.12 | 32.53 |
| Zelda | 23.44 | 26.52 | 27.97 | 30.09 | 32.44 |




**Graph 1 – RMSE to Compressed Image Size for images of dimension 512x512**

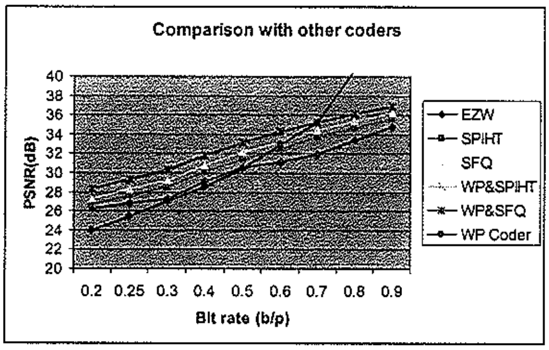



**Graph 2- Compression ratio to PSNR values for all images**



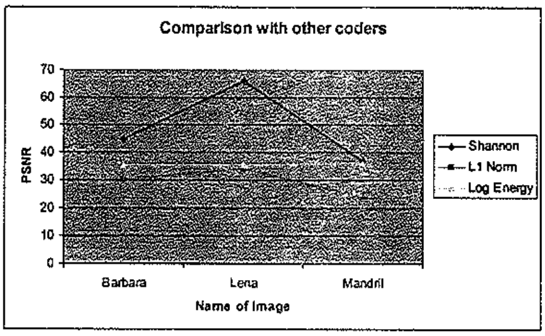

**Graph 3- PSNR values with bits per pixel for images of dimension 512 x 512**

From Graph 1 it is clear that as the RMSE value increases the size of the compressed image decreases. In case of graph 2 the PSNR value decreases as the compression ratio increases. Graph 3 plots PSNR values and compression ratios with different bits per pixel for each image. This graph indicates that as the bit rate increases the PSNR value increases. It is known that a higher compression ratio results in lower image quality for each individual image, so it is implied from this graph that as bit rate increases degradation of image quality is reduced.




**Graph 4- Comparison with other existing coders on 512 x 512 Barbara at different bit rates**

With the input image of 512 x 512 Barbara, a comparison of the wavelet packet coder implemented here is done with the other existing coders like-EZW Coder [8], the SPIHT Coder [7], the SFQ Coder [9], the WP & SPIHT Coder [5] and the WP & SFQ Coder [10], showing that the coder here lies below all the coders till the bit rate of 0.50 b/p after which the PSNR value increases. So it can be concluded that for higher bit rates the coder works well and out performs all the compared existing coders. All the wavelet packet coders compared here, except the WP & SPIHT Coder uses the rate-distortion method for finding the best basis from the wavelet packet tree. From the graph it is clear that the wavelet packet coders using the rate-distortion method perform better than all other coders, hence compressing using the rate-distortion method on the wavelet packets is efficient than using entropy based cost functions. It can also be concluded that compression using wavelet packets is more effective than compressing using wavelets. But while comparing the PSNR values with coders that does image compression using wavelet packets (Graph 5), using entropy cost functions such as -L1norm [3] and Shannon [4] for finding the best basis, it can be seen that the coder implemented here lies in between the Shannon entropy cost function and the L1 norm cost function, which implies that the Log energy cost function is better than L1 norm and worse with Shannon cost function.




**Graph 5- Comparison with other existing WP coders using different entropy cost functions**

## 6. Conclusion

A wavelet packet coder that allows compression of image using the log energy entropy cost function is presented here. An extensive analysis has been done using different images. This method has shown to achieve good quality image compression at high bit rates. Results have been compared with other existing coders showing that rate-distortion method is more effective than entropy based methods as using the entropy as a criterion for the best-basis selection can result into many coarse scale high frequency subbands. The number of decompositions is also a measure of quality. A larger number of decompositions can cause the loss of the coding algorithm efficiency, but as Human Visual System (HVS) is less sensitive to removal of smaller details, the decompressed image doesn't show much difference.

## References

[1] M.A. Cody, *"The Wavelet Packet Transform"*, Dr. Dobb's Journal, Vol 19, PP 44-46, 50-54, Apr. 1994.

[2] R. Coifman and M. V. Wickerhauser, *"Entropy-based algorithms for best basis selection"*, IEEE Trans. Inform. Theory, Vol. 38, PP 713–718, Mar. 1992.

[3] François G. Meyer, Amir Averbuch, Jan-Olov Strömberg and Ronald R. Coifman, *"Fast Wavelet Packet Image Compression"*, Data Compression Conference -DCC'98.

[4] G. K. Kharate and A. A. Ghatol and P.P. Rege, *"Image Compression Using Wavelet Packet Tree"*, ICGST International Journal on Graphics, Vision and Image Processing, Vol. 5, PP 37-40, 2005.

[5] Nikola Sprljan, Sonja Grgic and Mislav Grgic, *"Modified SPIHT algorithm for wavelet packet image coding"*, Real-Time Imaging, Vol. 11, Issue 5-6, PP 378-388, 2005.

[6] K.Ramachandran and M.Vetterli, *"Best wavelet packet bases in a rate-distortion sense"*, IEEE Trans.Image Proc., Vol.2, PP 160-175, April 1993.

[7] A. Said and W.A. Pearlman, *"A new fast and efficient image codec based on set partioning in hierarchical trees"*, IEEE Trans. on Cir. & Sys. for Video Tech., Vol. 6, PP 243–250, Jun. 1996.

[8] J.M. Shapiro, *"Embedded image coding using zerotrees of wavelet coefficients"*, IEEE Trans. on Signal Proc., Vol. 41, No. 12, PP 3445–3462, Dec. 1993.

[9] Z. Xiong, K. Ramchandran, and M. T. Orchard, *"Space-frequency quantization for wavelet image coding"*, IEEE Trans. Image Processing, Vol. 6, PP 677–693, May 1997.

[10] Z. Xiong, K. Ramchandran and M. T. Orchard, *"Wavelet Packet Image Coding Using Space-Frequency Quantization"*, IEEE Trans. Image Proc., Vol 7, PP 892-898, June 1998.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